Defn: a function $F$ is an antiderivative of $f$ on $(a, b)$ if $F'(x) = f(x)$ on $(a, b)$.

[Note that this defined locally.]

Given an antiderivative of $f$, we may form the general antiderivative:

$$F(x) + C$$

where $C$ is an arbitrary constant.

This general antiderivative is often denoted by $\int f(x)\,dx$. 
a specific antiderivative satisfies some additional initial conditions.

Define: a function \( F \) is the specific antiderivative of \( f \) with initial conditions of \( F \) satisfies

\[
F'(x) = f(x)
\]

and

\[
F(a) = b
\]

for some \( a, b \in \mathbb{R} \).

(e.g.) Find the specific antiderivative \( F \) of \( f(x) = 2x \).

\[
F(0) = 1
\]

Clearly \( F(x) = x^2 + C \) is a general antiderivative; then we additionally require

\[
1 = F(0) = (0)^2 + C \implies C = 1
\]

hence \( \int F(x) = x^2 + 1 \)
Note that antiderivatives follow the linearity rules:

If $F$ is an antiderivative of $f$ and $G$ is an antiderivative of $g$

then $aF + bg$ is an antiderivative of $af + bg$.

In $\int$ notation:

$$\int (af + bg) \, dx = a \int f \, dx + b \int g \, dx$$

The rules for the basic antiderivatives are simple inverses of our first derivative rules:

- **Power** ($n \neq -1$)
  \[
  \int x^n \, dx = \frac{x^{n+1}}{n+1} + C
  \]

- **Exponential**
  \[
  \int e^x \, dx = e^x + C
  \]

- **Trig**:
  \[
  \int \sin(x) \, dx = -\cos(x) + C
  \]
  \[
  \int \cos(x) \, dx = \sin(x) + C
  \]
Note a new one (that may be obvious given the derivative of \( \ln(x) \)):

\[
\int \frac{1}{x} \, dx = \ln|x| + C
\]

\[\rightarrow\] Her scalar multiples of the argument \( x \) use the Chain Rule in reverse:

If \( F(x) \) is an antiderivative of \( f(x) \), then

\[
\frac{1}{a} F(ax) \quad \text{is an antiderivative of} \quad f(ax)
\]