Appendix 3 Language of sets

- A set is a collection of objects (usually numbers or pairs of numbers for this class).

\[ S = \{1, 2, 3, 4, 5\} \quad  N = \{0, 1, 2, \ldots \} \]
\[ T = \{1, 2\} \quad K = \{2^0, 2^1, 2^2, \ldots \} \]
\[ E = \{2, 4, 6, 8\} \quad \text{etc.} \]

- A member of a set is called an element and we express this using \(\in\):

\[ \begin{align*}
2 &\in S \quad 0 \in N \quad 255 \in N \\
1 &\in T \quad 2^{10} \in K
\end{align*} \]

- A set contained in another set is a subset and we express this using \(\subseteq\):

\[ \begin{align*}
T &\subseteq S \quad S \subseteq N \\
E &\subseteq N
\end{align*} \]
- Intersection:
  \[ S \cap E = \text{all elements in } S \text{ and } E \]
  \[ = \{2, 4\} \]

- Union:
  \[ S \cup E = \text{all elements in } S \text{ or } E \]
  \[ = \{1, 2, 3, 4, 6, 8\} \]

- Language of set construction in IR:
  "\[ S = \{x \in \mathbb{R} \mid \text{property } P\} \]" can be read as
  "\[ S \text{ is the set of all } x \text{ in } \mathbb{R} \text{ such that } x \text{ satisfies property } P \]"

- Examples: the intervals:
  \[ [a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\} \]
  \[ (a, b) = \{x \in \mathbb{R} \mid a < x < b\} \]
* number systems:

\[ \mathbb{N} = \{0, 1, 2, 3, \ldots\} \quad \text{natural numbers} \]

\[ \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \quad \text{integers} \]

\[ \mathbb{Q} = \{x \in \mathbb{R} \mid x = \frac{p}{q} \text{ for some } p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}^+\} \quad \text{rational numbers} \]

\[ \mathbb{R} = \text{the real line} \]

* graphs: the graph of a function \( f \)

is a subset of the Cartesian coordinate plane: \( \mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\} \)

given by:

\[ \text{graph}(f) = \{(x, f(x)) \mid x \in \mathbb{R}\} \]