Exam rules:

1. No electronic devices of any kind are allowed during this exam.

2. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam.

3. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions:

1. Write your first and last name at the top of every page of this exam.

2. Write your solutions clearly and legibly. No credit will be given for illegible solutions.

3. Show the work leading to your solutions. Partial credit can only be given based on the work you show.

4. Please put a box around your final answer to each part.

→ If any question is not clear, ask for clarification.
1. (10 points) Consider the following two curves:

\[ y = \frac{6}{x} \quad \text{and} \quad y = 5 - x \]

Find the area of the region between the two curves.
2. (10 points) Setup, but **do not evaluate**, an integral giving the volume of the solid obtained by revolving the region in question 1 about the $x$-axis.
3. (10 points) Find the general antiderivative of the following function using integration by parts:

\[ f(x) = x^2 \cos(3x) \]
4. (10 points) Find the area contained inside 3 leaves of the curve given by the polar graph:

\[ r = 2 \cos(5\theta) \]
5. (10 points) Find the general antiderivative of the following product of trigonometric functions:

\[ f(x) = \cos^2(x) \sin^2(x) \]
6. (10 points) Consider the following rational function:

\[ f(x) = \frac{3x^2 + 5x + 18}{(x^2 + 16)(x + 2)} \]

(a) Find the partial fraction decomposition of \( f(x) \) over the real numbers.
(b) Evaluate the indefinite integral $\int f(x)dx$ using your answer to part (a).
EXTRA CREDIT (+5 points): Find the partial fraction decomposition of $f(x)$ over the complex numbers. The complex numbers in your answer must be in rectangular form, i.e. as $x + iy$. 
7. (10 points)

(a) Determine (directly) whether the following improper integral is convergent or divergent:

\[ \int_{1}^{\infty} \frac{2x \, dx}{(3x^2 + 5)^5} \]

If the integral is convergent, give its value.
(b) Determine (by comparison) whether the following improper integral is convergent or divergent:

\[ \int_{1}^{\infty} \frac{dx}{(3x^2 + 5)^5} \]

You must justify your answer, but you do not need to give the value if it is convergent.
8. (10 points) Consider the following sequence:

\[ a_n = \frac{2^{n+1} 3^n}{\pi^{2n+1}} \]

(a) Find the limit of the sequence: \( \lim_{n \to \infty} a_n \)

(b) Determine whether the associated series is convergent or divergent:

\[ \sum_{n=0}^{\infty} a_n = a_0 + a_1 + a_2 + \ldots \]
(c) Determine whether the following series is convergent or divergent:

\[
\sum_{n=0}^{\infty} \frac{2^{n+1}3^n}{\pi^{2n+1}(n + 1)!}
\]

You must justify your answer, but you do not need to give the value if it is convergent.
9. (10 points) Determine the interval of convergence for the following power series:

\[ F(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n+1} (x + 1)^n \]
10. (10 points) Find the Taylor series for the following function:

\[ f(x) = \frac{x^3}{1 + 2x} \]