Exam rules:

1. No electronic devices of any kind are allowed during this exam.

2. You may use one 2-sided US Letter sized page of notes, but no books or other assistance are allowed during this exam.

3. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Instructions:

1. Write your first and last name at the top of every page of this exam.

2. Write your solutions clearly and legibly. No credit will be given for illegible solutions.

3. Show the work leading to your solutions. Partial credit can only be given based on the work you show.

4. Please put a box around your final answer to each part.

→ If any question is not clear, ask for clarification.
1. (10 points) Consider the following indefinite integral:

\[ \int x^3 \sqrt{x^2 - 16} \, dx \]

(a) Rewrite the indefinite integral using the following trigonometric substitution:

\[ x = 4 \sec(t) \]

(Your answer will be a trigonometric integral.)

(b) Evaluate the trigonometric integral found in part (a).
(c) Evaluate the given indefinite integral using your answer from part (b) and the substitution from part (a). (That is, “re-substitute” to evaluate the given indefinite integral in terms of \( x \).)
2. (10 points) Consider the rational function:

\[ f(x) = \frac{6x^2 - 19x + 20}{(x + 1)(x - 2)^2} \]

(a) Find the partial fraction decomposition of \( f(x) \) over the real numbers.
(b) Evaluate the indefinite integral $\int f(x) \, dx$ using your answer to part (a).
3. (10 points)

(a) Determine (directly) whether the following improper integral is convergent or divergent:

\[ \int_{1}^{\infty} \frac{x^3 \, dx}{5x^4 + 3} \]

If the integral is convergent, give its value.
(b) Determine (by comparison) whether the following improper integral is convergent or divergent:

\[ \int_{1}^{\infty} \frac{(x^3 + x^2 + x + 1)}{5x^4 + 3} \, dx \]

You must justify your answer, but you do not need to give the value if it is convergent.
4. (10 points) Consider the following sequence:

\[ a_n = \frac{3n}{2n^2 - 1} \]

(a) Find the limit of the sequence: \( \lim_{n \to \infty} a_n \)

(b) Determine whether the associated series is convergent or divergent:

\[ \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots \]

If the series is convergent, give its value.
(c) Determine whether the following series is convergent or divergent:

\[ \sum_{n=0}^{\infty} \frac{3n + 1}{2n^2 - n} \]

You must justify your answer, but you do not need to give the value if it is convergent.