

Math 170A: Numerical Linear Algebra

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Sketch of Solution for Homework 2

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1. (2.1.14)

For $p = 1, 2, \infty$ you can see at [http://en.wikipedia.org/wiki/Norm_\(mathematics\)](http://en.wikipedia.org/wiki/Norm_(mathematics)). For other values of p , you can write one coordinate in term of function of the other and plot the function in Matlab, e.g. $p = 3/2$, $x_2 = (1 - |x_1|^{\frac{3}{2}})^{\frac{2}{3}}$

2. (2.1.17)

(a)

$$\|x\|_A^2 = x^T A x = x^T R^T R x = (R x)^T (R x) = \|R x\|_2$$

(b) Use the identity from a) prove that $\|\cdot\|_A$ has the 3 properties on p.112.

3. (2.1.23)

A counter example is as following

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

4. (2.1.31) These inequalities can be proved using definitions. Only the middle one need other stuff (Cauchy-Schwartz Inequality (C-S), see p.113)

- $\|x\|_\infty^2 = x_{i_0}^2$, for some $1 \leq i_0 \leq n$
 $\|x\|_2^2 = \sum_1^n x_i^2$
- $\|x\|_1^2 = \|x\|_2^2 +$ nonnegative cross terms.
- by C-S, $(\sum_1^n |x_i|)^2 \leq \sum_1^n 1^2 \sum_1^n x_i^2$.
- Use $|x_i| \leq \|x\|_\infty$, for all $i = 1, \dots, n$

5. (2.1.32)

Use definition of matrix norm and inequalities from (2.1.31), e.g.

$$\|A\|_{\infty} = \max_{x \neq 0} \frac{\|Ax\|_{\infty}}{\|x\|_{\infty}} \leq \max_{x \neq 0} \frac{\|Ax\|_2}{\frac{1}{\sqrt{n}}\|x\|_2}$$

The top use the first inequality and the bottom use the last one from (2.1.31).

Other inequalities for this problem can be carried out in the same way.

6. (2.2.6)

Straight forward when plug in the definitions.

7. (2.2.23)

- Find x in the direction of $\text{maxmag}(A)$ and δb in the direction of $\text{maxmag}(A^{-1})$

$$b = \begin{pmatrix} 749 \\ 1502 \end{pmatrix}, \delta b = \begin{pmatrix} -.01 \\ .01 \end{pmatrix}, x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \delta x = \begin{pmatrix} -5.62 \\ 5.64 \end{pmatrix}$$

- Find b in the direction of $\text{maxmag}(A^{-1})$ and δx in the direction of $\text{maxmag}(A)$

$$b = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \delta b = \begin{pmatrix} 7.49 \\ 15.02 \end{pmatrix}, x = \begin{pmatrix} -562 \\ 563.5 \end{pmatrix}, \delta x = \begin{pmatrix} .01 \\ .01 \end{pmatrix}$$