$\boxed{1.7.32}$ Only need to $\mathrm{diag}(D) > 0$ (why?).

Suppose $d_{ii} \le 0$. Let $x = L^{-T} y$, where $y = \begin{array}{c} \\ \\ \leftarrow i^{th} \\ \\ \end{array}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ (why L^{-T} exists?).

Since L^{-T} nonsingular (why?), $y \neq 0 \Rightarrow x = L^{-T} y \neq 0$.

From A is positive def. $\Rightarrow 0 < x^T A x \underset{(why?)}{=} y^T D y = d_{ii} \le 0$

$\Rightarrow$ contradiction.

$\boxed{1.7.34}$

a) Just do the multiplication MA and look at M row by row,

b) $\det(M) = 1$ obvious since M is ~~low~~ unit lower triangular.

$\hat{A} = MA \Rightarrow \det(\hat{A}) = \det(M) \cdot \det(A)$

c) Let N be the matrix which is different from M only in that it has $-m$ instead of m in the (i,j). Use result from a) show $MN = I$

$\Rightarrow N = M^{-1}$

$\boxed{1.7.44}$ Follow step by step the outline given in the problem.

a) Follow step by step the outline given in the problem.

b) $L^{-1} \cdot L = I$. $\quad (L^{-1} = (\tilde{l}_{ij})_{n\times n})$

Look at component (i,i) of both side: $\sum_{k=1}^{n} \tilde{l}_{ik} l_{ki} = 1 \ (*)$

However: $l_{ki} = 0$ when $k < i$ $\quad (L^{-1}, L$ are lower $\triangle)$.

$\tilde{l}_{ik} = 0$ when $k > i$

$(*)$ becomes: $\quad \tilde{l}_{ii} \cdot l_{ii} = 1 \Rightarrow \tilde{l}_{ii} = l_{ii}^{-1}$

$\boxed{1.7.45}$

Let $A = LM$, $\quad a_{ij} = \sum_{k=1}^{n} l_{ik} m_{kj}$

Since L, M are lower triangular: $\quad l_{ik} = 0$ if $k > i$

$m_{kj} = 0$ if

For $i<j$, either $l_{ik}=0$ or $m_{kj}=0$ $\forall k$.

$\Rightarrow a_{ij}=0$ for $i<j$ $\Rightarrow$ A is $\triangle$

b) On the diagonal.

$$a_{ii} = \sum l_{ik}\cdot m_{ki} \underset{\uparrow}{=} l_{ii}\cdot m_{ii}$$

use the same argument in a).

$\boxed{1.7.50}$

a) multiply out and compare both sides.

b) Plug in M and $\tilde{A}_{22}$ in and do multiplication to show LHS = RHS

c) Look at 1.7.52 but in a different way of partition matrices in blocks.

$$\left[\begin{array}{cc|c} A_{11}^{(k\times k)} & * & \\ * & A_{22}^{(j\times j)} & * \\ \hline & * & A_{22}^{(l-j)\times(l-j)} \end{array}\right] = \left[\begin{array}{cc|c} I_k & O & O \\ * & I_j & \\ \hline & * & I_{l-j} \end{array}\right] * \left[\begin{array}{cc|c} A_{11}^{k\times k} & * & * \\ O & \tilde{A}_{22}^{j\times j} & \\ \hline & O & A_{22}^{(l-j)} \end{array}\right]$$

$$\Rightarrow \underbrace{\begin{bmatrix} A_{11}^{k\times k} & * \\ * & A_{22}^{j\times j} \end{bmatrix}}_{} = \begin{bmatrix} I_k & O \\ * & I_J \end{bmatrix} * \begin{bmatrix} A_{11}^{k\times k} & * \\ O & \tilde{A}_{22}^{j\times j} \end{bmatrix}$$

$(k+j)^{th}$ leading principal submatrix of A, call $\vec{A}$

Look at det of both sides $\Rightarrow \underset{\neq 0}{\det(\tilde{A})} = \underset{\neq 0}{\det(A_{11})}\cdot\det(\tilde{A}_{22}^{j\times j})$

~~since~~ $\Rightarrow \det(\tilde{A}_{22}^{j\times j}) \neq 0 \Rightarrow$ n.s

This true for $\forall$ $1\leq j\leq l$.