2.6.2)
\[ \int_{t}^{t+T} f(x) \, dx = \int_{x(t)}^{x(t+T)} f(x) \, dx \]

By the chain rule [1 point]

Since \( x(t) = x(t + T) \Rightarrow \int_{x(t)}^{x(t+T)} f(x) \, dx = 0 \) [1 point for conclusion]

\[ \int_{t}^{t+T} f(x) \, dx = \int_{t}^{t+T} \frac{dx}{dt} \, dt = \int_{t}^{t+T} \left( \frac{dx}{dt} \right)^2 \, dt \] [1 point]

Since \( \left( \frac{dx}{dt} \right)^2 \geq 0 \), \( T > 0 \) and \( x(t) \) is a nontrivial solution \( \Rightarrow \int_{t}^{t+T} \left( \frac{dx}{dt} \right)^2 \, dt > 0 \), a contradiction [1 point for conclusion]

2.7.1)

\[-\frac{dV}{dx} = \frac{dx}{dt} = x(1-x)\]

Find equilibrium points first: \( \Rightarrow x(1-x) = 0 \Rightarrow x = 0 \) and \( x = 1 \) are equilibrium points [2 points, 1 for each point]

(Can also identify them graphically)

Find the potential \( V(x) \): \( -\frac{dV}{dx} = x(1-x) \Rightarrow \int dv = \int x(x-1) \, dx \Rightarrow V(x) = \frac{x^3}{3} - \frac{x^2}{2} + C \) [1 point for correct potential]

Set \( C = 0 \) and plot \( V(x) \) vs \( x \): (If a different \( C \) is chosen the plot will be shifted up or down) [1 point]

![Graph of V(x)](image)

It can be seen that \( x = 0 \) is a local maximum and \( x = 1 \) is a local minimum, so \( x = 0 \) is an unstable equilibrium point and \( x = 1 \) is an stable equilibrium point [1 point for correct conclusion]

3.1.1)

\[ \frac{dx}{dt} = 1 + rx + x^2 \]

Plots Left to right: for \( r = 1 \), for \( r = -2 \) and bottom for \( r = 4 \) [3 points, 1 for each plot]
Can also have a stack of vector fields showing no equilibrium points for \(-2 < r < 2\), one equilibrium point for \(r = 2, -2\) and two equilibrium points for \(|r| > 2\)

\[
\frac{dx}{dt} = 1 + rx + x^2 = 0 \Rightarrow x = \frac{-r \pm \sqrt{r^2 - 4}}{2} \quad [1 \text{ point for right answer}]
\]