6.6.1 If $\tilde{x}(t) = x(-t)$ and $\tilde{y}(t) = -y(-t)$, then [1 pt]

$$\frac{d}{dt}\tilde{x}(t) = -\dot{x}(-t) = -y(-t)(1 - x(-t)^2) = \tilde{y}(t)(1-\tilde{x}(t)^2), \quad [1 \text{ pt}]$$

and

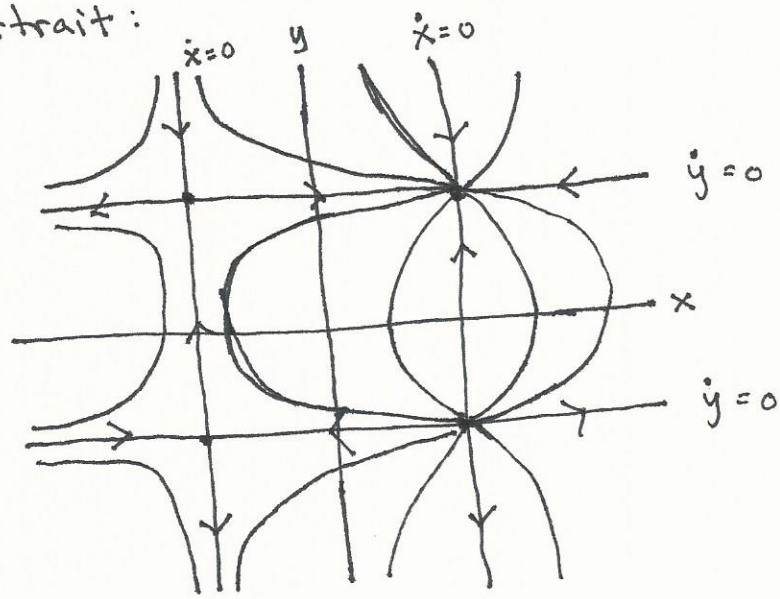
$$\frac{d}{dt}\tilde{y}(t) = \dot{y}(-t) = 1 - y(-t)^2 = 1 - \tilde{y}(t)^2. \quad [1 \text{ pt}]$$

This shows that the system is invariant under $t \mapsto -t$ and $y \mapsto -y$, so it's reversible.

Fixed pts are $(\pm 1, \pm 1)$.

LSA says that
- $(1,1)$ will be a stable star
- $(1,-1)$ will be an unstable star
- $(-1,1)$ and $(-1,-1)$ will be saddles

Phase portrait:



[2 pts]

6.6.7 If $\tilde{x}(t) = -x(-t)$ and $\tilde{y}(t) = y(-t)$, then [1 pt]

$$\frac{d}{dt}\tilde{x}(t) = \dot{x}(-t) = y(-t) = \tilde{y}(t), \quad [1 \text{ pt}]$$

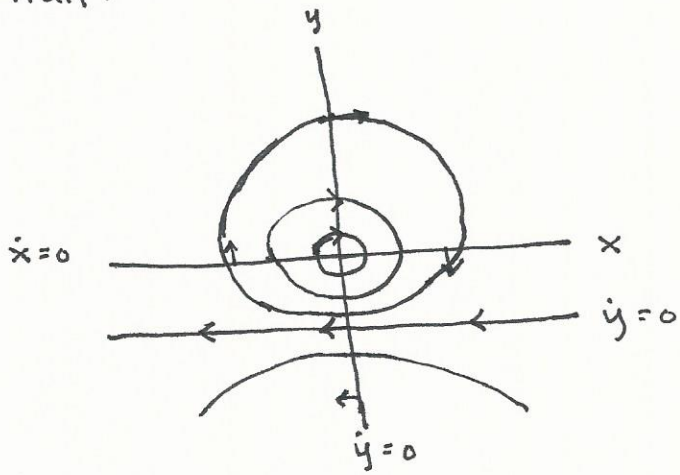
and

$$\frac{d}{dt}\tilde{y}(t) = -\dot{y}(-t) = x(-t)(1+y(-t)) = -\tilde{x}(t)(1+\tilde{y}(t)). \quad [1 \text{ pt}]$$

This shows that the system is invariant under $t \mapsto -t$ and $x \mapsto -x$, so it's reversible.

Fixed point is $x=0, y=0$. LSA says that $(0,0)$ will be a center. Since the system is reversible, it will be a center.

Phase portrait:



[2 pts]

6.7.1 $\dot{\theta} = \nu$

$\dot{\nu} = -b\nu - \sin\theta$

Fixed pts $(\theta, \nu) = (k\pi, 0)$ for $k \in \mathbb{Z}$.

$J = \begin{pmatrix} 0 & 1 \\ -\cos\theta & -b \end{pmatrix}$; $\tau = -b$ $\Delta = \cos\theta = (-1)^k$ when $\theta = k\pi$.

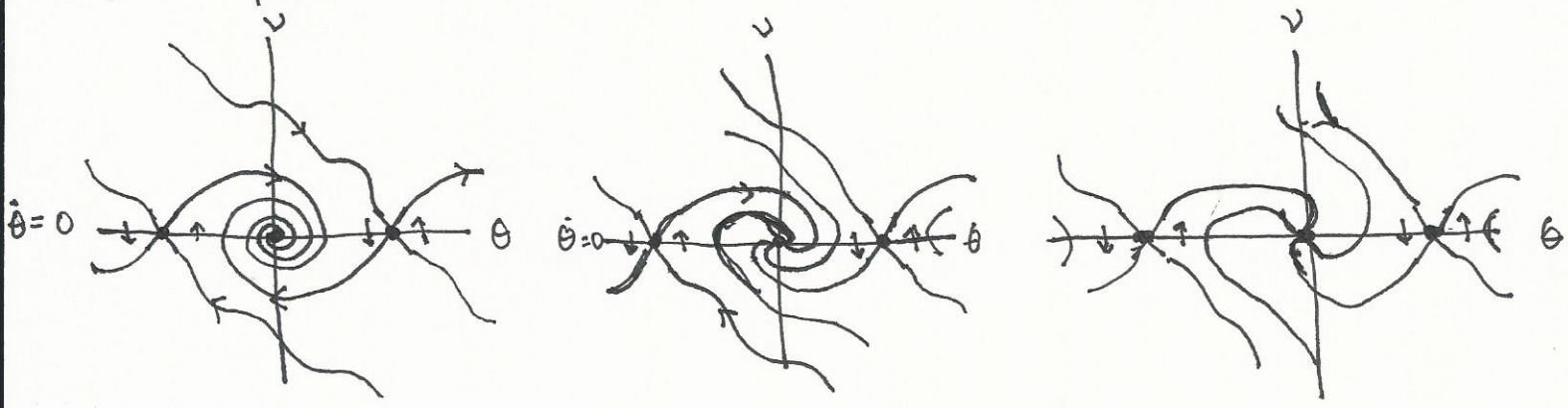
So $\tau^2 - 4\Delta = b^2 - 4(-1)^k$ when $\theta = k\pi$ and $\nu = 0$.

If k is odd then $(k\pi, 0)$ will be a saddle.

If k is even, then $0 < b < 2 \Rightarrow (k\pi, 0)$ will be a stable spiral, $b = 2 \Rightarrow (k\pi, 0)$ will be a degenerate node and $b > 2 \Rightarrow (k\pi, 0)$ will be a stable node.

[2 pts]

phase portrait:



$0 < b < 2$ $b = 2$ $b > 2$

[1 pt for each correct phase portrait]