6. [6.] If \( \dot{x}(t) = x(-t) \) and \( \dot{y}(t) = -y(-t) \), then

\[
\frac{d}{dt} \dot{x}(t) = -\dot{x}(-t) = -y(-t) (1 - x(-t^3)) = \dot{y}(t) (1 - \dot{x}(t)^2),
\]

and

\[
\frac{d}{dt} \dot{y}(t) = \dot{y}(-t) = 1 - y(-t)^2 = 1 - \dot{y}(t)^2.
\]

This shows that the system is invariant under \( t \rightarrow -t \) and \( y \rightarrow -y \), so it's reversible.

Fixed pts are \((\pm 1, \pm 1)\).

LSA says that:
- \((1, 1)\) will be a stable star
- \((1, -1)\) will be an unstable star
- \((-1, 1)\) and \((-1, -1)\) will be saddles

Phase portrait:
If \( \tilde{x}(t) = -x(-t) \) and \( \tilde{y}(t) = y(-t) \), then

\[
\frac{d}{dt} \tilde{x}(t) = \dot{x}(-t) = y(-t) = \tilde{y}(t),
\]

and

\[
\frac{d}{dt} \tilde{y}(t) = -\dot{y}(-t) = x(-t)(1 + y(-t)) = -\tilde{x}(t)(1 + \tilde{y}(t)).
\]

This shows that the system is invariant under \( t \rightarrow -t \) and \( x \rightarrow -x \), so it's reversible.

Fixed point is \( x=0, y=0 \). LSA says that \((0,0)\) will be a center. Since the system is reversible, it will be a center.

Phase portrait:
Fixed pts $(\theta, v) = (k\pi, 0)$ for $k \in \mathbb{Z}$.

$J = \begin{pmatrix} 0 & 1 \\ -\cos \theta & -b \end{pmatrix}$; $\Delta = -b$ $\Delta = \cos \theta = (-1)^k$ when $\theta = k\pi$.

So $\Delta^2 - 4\Delta = b^2 - 4(-1)^k$ when $\theta = k\pi$ and $v = 0$.

If $k$ is odd then $(k\pi, 0)$ will be a saddle.

If $k$ is even then $0 < b < 2 \Rightarrow (k\pi, 0)$ will be a stable spiral; $b = 2 \Rightarrow (k\pi, 0)$ will be a degenerate node and $b > 2 \Rightarrow (k\pi, 0)$ will be a stable node.

phase portrait:

0 < $b$ < 2

$b = 2$

$b > 2$

[1 pt for each correct phase portrait]