7.1.3: \[ \dot{i} = \lambda (\frac{1}{-\lambda})(4-\lambda) \]
\[ \dot{\lambda} = 2 - \lambda^2 \]

First, we can plot \( i \) and \( \dot{\lambda} \) in function of \( \lambda \):

From this (or directly from the equations) we get that:

1. \( i > 0 \iff \lambda \in (0,1) \cup (2,\infty) \) - here trajectories move outward
   - \( i < 0 \iff \lambda \in (1,2) \) - inward
   - \( i = 0 \iff \lambda \in \{0,1,2\} \)

2. \( \dot{\lambda} > 0 \iff \lambda \in [0,\sqrt{2}) \) - trajectories move counterclockwise
   - \( \dot{\lambda} < 0 \iff \lambda \in (\sqrt{2},\infty) \) - clockwise
   - \( \dot{\lambda} = 0 \iff \lambda = \pm \sqrt{2} \)

We see that \((0,0)\) is a fixed point. Also the circles \( \lambda = 1 \) and \( \lambda = 2 \) will be limit cycles since \( \dot{\lambda} \neq 0 \) for \( \lambda = 1,2 \) and \( i \neq 0 \) for \( \lambda \neq 0,1,2 \). We get:
7.1.8 \[ \dot{x} + a x (x^2 + x^2 - 1) + x = 0, \ a > 0 \]
a) We rewrite the system:
\[
\begin{cases}
\dot{x} = y \\
\dot{y} = -ay (x^2 + y^2 - 1) - x
\end{cases}
\]
Putting both equal to zero, we find as only fixed point \((x^*, y^*) = (0, 0)\).
The Jacobian at \((0, 0)\) is \(\begin{pmatrix} 0 & 1 \\ -1 & a \end{pmatrix}\), in particular \(\tau = a, \ \Delta = 1\).

From chapters 5 & 6:
- if \(a < 2\): unstable spiral
- \(a = 2\): unstable node
- \(a > 2\): unstable (but not sure which one, as this is a borderline case).

b) Changing to polar coordinates, one gets
\[
i = \frac{x \dot{x} + y \dot{y}}{\sqrt{x^2 + y^2}} = -a \sin^2 \theta (x^2 - 1)
\]
\[
\dot{\theta} = \frac{x \dot{y} - y \dot{x}}{\sqrt{x^2 + y^2}} = -a \cos \theta \sin \theta (x^2 - 1) - 1
\]
So for \(a = 1\), we get \(\dot{i} = 0, \ \dot{\theta} = -1\). Hence the circle \(\tau = 1\) is a limit cycle with amplitude 1 and period \(T = \int_0^{2\pi} \frac{1}{\theta} \, d\theta = \int_{-1}^{1} d\theta = -2\pi\), i.e., period \(2\pi\) in clockwise direction.

c) Since \(\dot{i} > 0\) if \(\tau < 1\), the limit cycle is stable.
\[
i < 0 \text{ if } \tau > 1
\]
d) Same as c1: if \(\tau < 1\), the radius always increases and if \(\tau > 1\), the radius always decreases, so there are no possibilities for other limit cycles.

7.2.8: Suppose \(\begin{cases} \dot{x} = \frac{\partial V}{\partial x} \\ \dot{y} = \frac{\partial V}{\partial y} \end{cases}\) is a gradient system. Since \(V\) is a differentiable function of two variables, its equipotentials are differentiable paths.
Suppose \((\bar{x}(H), \bar{y}(H))\) is such an equipotential, i.e., \(V(\bar{x}(H), \bar{y}(H)) = c\).
Differentiating we get by the chain rule
\[
0 = \frac{d}{dH} V(\bar{x}(H), \bar{y}(H)) = \frac{\partial V}{\partial x} \ddot{x} + \frac{\partial V}{\partial y} \ddot{y} = -\ddot{x} - \ddot{y} \dddot{x} \quad \text{and so the inner product between the tangent to the trajectory} \ (x, y) \ \text{and the equipotential} \ (x, y) \ \text{is 0, i.e. they cross orthogonally.} \]