9.4.2 a)

b) Fixed points satisfy either $2x = x$ or $2 - 2x = x$. This gives us the fixed points $x^* = 0$ and $x^* = \frac{2}{3}$. As $|f'(x^*)| = 2 > 1$ for both, they are unstable.

c) One sees that $f(f(x)) = \begin{cases} 4x & \text{if } 0 \leq x \leq \frac{1}{4} \\ 2 - 4x & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 4x - 2 & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 4 - 4x & \frac{3}{4} \leq x \leq 1 \end{cases}$

Fixed points for $f^{(2)}$ are then easily calculated to be $x^* = 0, \frac{2}{5}, \frac{2}{3}, \frac{4}{5}$. Since 0 and $\frac{2}{3}$ are fixed points of $f$, they don't give period-2 orbits. On the other hand, we have $f(\frac{2}{5}) = \frac{4}{5}, f(\frac{4}{5}) = \frac{2}{5}$, which gives us a period-2 orbit. Since $|f'(x)| > 1$ for all $x$, the orbit is unstable.

(OR: the multiplier is $\lambda = |f'(x)(f'(x))| = 2 > 1$.)

10.1.11: a) Solving $3x - x^3 = x$, we get the fixed points $x^* = 0, \pm \sqrt{2}$. Moreover $|f'(x)| = |f'(\pm \sqrt{2})| = 3 > 1$, so they are unstable.
d) For \( f(x) = 3x - x^3 \), we see that \( f([-2, 2]) \subseteq [-2, 2] \). Hence an orbit starting in \([-2, 2]\) will stay inside this region. For \(|x| > 2\) on the other hand, we have \(|f'(x)| > 1\), so orbits will escape to \( \infty \).

10.3.1: We need a superstable fixed point, i.e. \( x^* \) such that \( f(x^*) = x^* \) and \( f'(x^*) = 0 \).

Plugging in gives the following:

\[
\begin{align*}
 f(x^*) &= x^* \\
 f'(x^*) &= 0
\end{align*}
\]

\(-r x(1-x) = x^* \Rightarrow x^* = 0, \quad r = 1\)

\(-r(1-2x^*) = 0 \Rightarrow x^* = \frac{1}{2} \quad \text{or} \quad r = 0.\)

For \( r = 0 \), we get the uninteresting case \( f = 0 \) (and \( 0 \) is then obviously a superstable fixed point).

Otherwise we need \( x^* = \frac{1}{2} = \frac{r-1}{r} \) and so \( r = 2 \).