BOB

Homework 8

9.4.2 a)

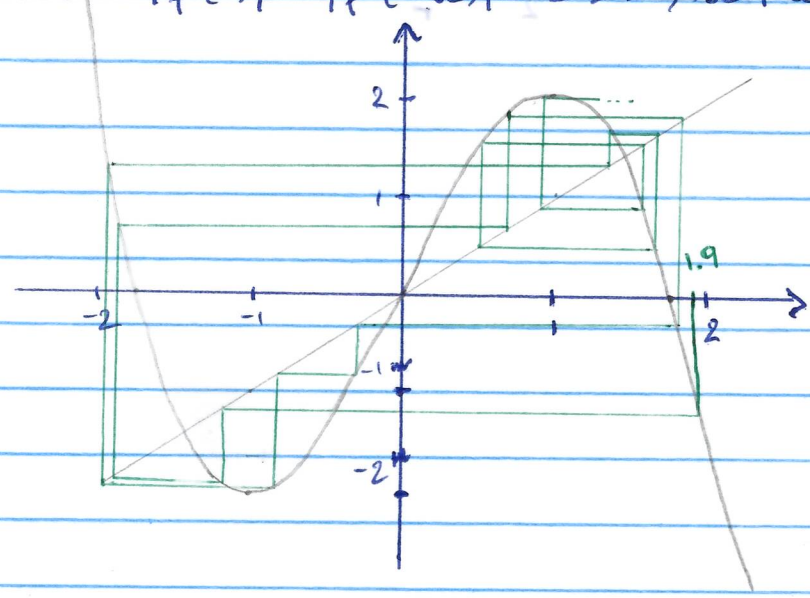
b) Fixed points satisfy either $2x = x$ or $2-2x = x$. This gives us the fixed points $x^* = 0$ and $x^* = \frac{2}{3}$. As $|f'(x^*)| = 2 > 1$ for both, they are unstable.

c) One sees that
$$f(f(x)) = \begin{cases} 4x & \text{if } 0 \le x \le \frac{1}{4} \\ 2-4x & \frac{1}{4} \le x \le \frac{1}{2} \\ 4x-2 & \frac{1}{2} \le x \le \frac{3}{4} \\ 4-4x & \frac{3}{4} \le x \le 1 \end{cases}$$

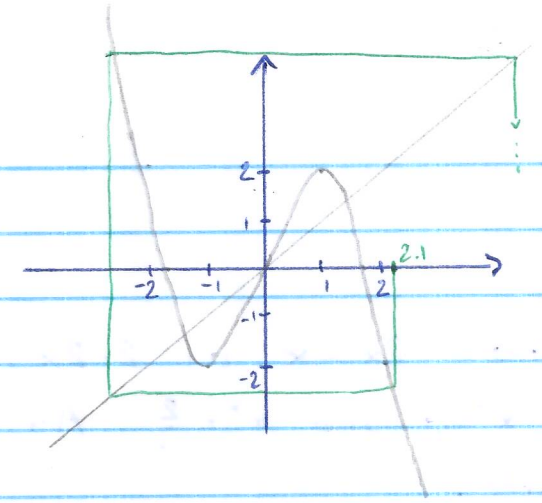
Fixed points for $f^{(2)}$ are then easily calculated to be $x^* = 0, \frac{2}{5}, \frac{2}{3}, \frac{4}{5}$. Since 0 and $\frac{2}{3}$ are fixed points of f, they don't give period-2 orbits. On the other hand, we have $f(2/5) = 4/5$, $f(4/5) = 2/5$, which gives us a period-2 orbit. Since $|f'(x)| > 1$ for all x, the orbit is unstable.
(OR: the multiplier is $\lambda = |f'(p)||f'(q)| = 2 \cdot 2 = 4 > 1$.)

10.1.11: a) Solving $\overset{f(x)}{3x - x^3} = x$, we get the fixed points $x^* = 0, \pm\sqrt{2}$. Moreover $|f'(0)| = |f'(\pm\sqrt{2})| = 3 > 1$, so they are unstable.

b)



c)



d) For $f(x) = 3x - x^3$, we see that $f([-2,2]) \subseteq [-2,2]$. Hence an orbit starting in $[-2,2]$, will stay inside this region.
For $|x| > 2$ on the other hand, we have $|f'(x)| > 1$, so orbits will escape to ∞.

<u>10.3.1</u>: We need a superstable fixed point, i.e. x^* such that $f(x^*) = x^*$ and $f'(x^*) = 0$.
Plugging in gives the following:

$$f(x^*) = x^* \Rightarrow rx^*(1-x^*) = x^* \Rightarrow x^* = 0, \frac{r-1}{r}$$

$$f'(x^*) = 0 \Rightarrow r - 2rx^* = 0 \Rightarrow x^* = \frac{1}{2} \text{ or } r = 0.$$

For $r = 0$, we get the uninteresting case $f = 0$ (and 0 is then obviously a superstable fixed point).
Otherwise we need $x^* = \frac{1}{2} = \frac{r-1}{r}$ and so $r = 2$.