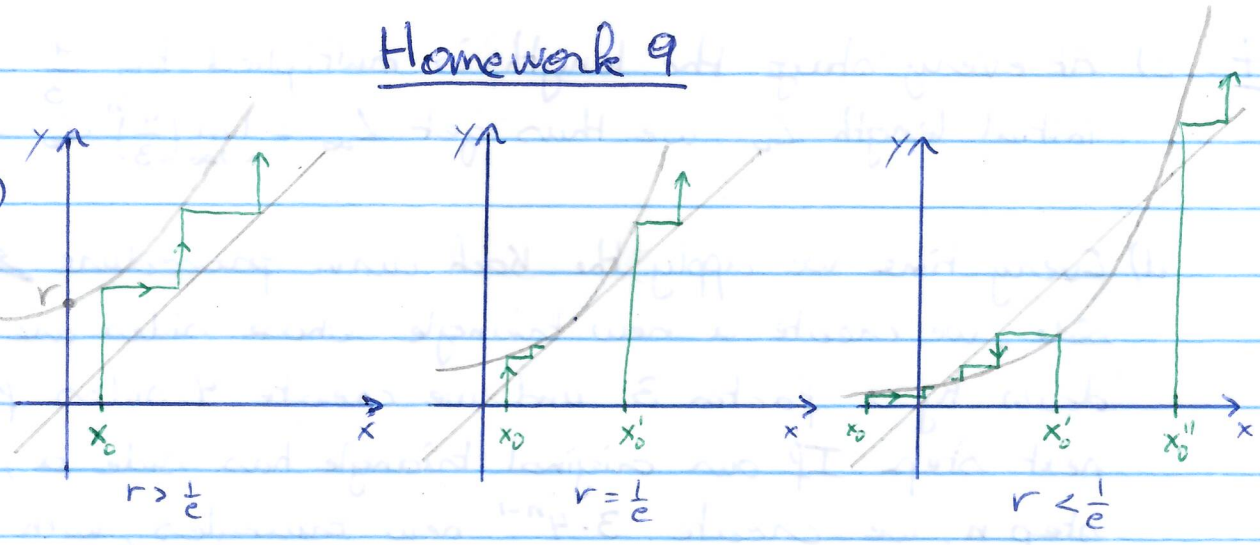
B0B

Homework 9

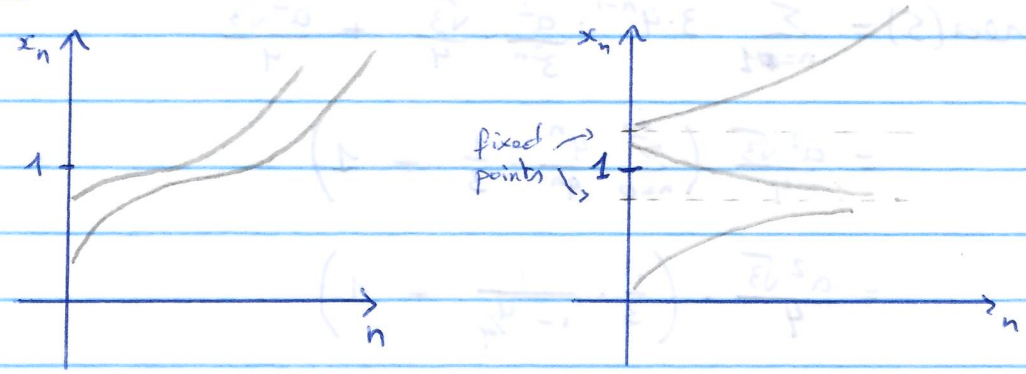
10.4.1 a)



b) At a tangent bifurcation, we will have

$$\begin{cases} re^x = x \\ \frac{d}{dx} re^x = \frac{d}{dx} x \end{cases} \Rightarrow \begin{cases} re^x = x \\ re^x = 1 \end{cases} \Rightarrow x = 1 \text{ and } r = \frac{1}{e}$$

c)



$r \gtrsim \frac{1}{e}$ $\qquad$ $r \lesssim \frac{1}{e}$

10.5.1: $f(x) = rx \Rightarrow f'(x) = r$.

Plugging into the definition of the Liapunov exponent, we get

$$\lambda = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=0}^{n-1} \ln|f'(x_i)|\right) = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=0}^{n-1} \ln|r|\right) = \ln|r|.$$

11.3.1 a) The middle half Cantor set consists of 2 copies of itself, each scaled down by a factor 4. Hence its self similarity dimension is $\frac{\ln(2)}{\ln(4)} = \frac{1}{2}$.

b) Removing the middle half of a line segment, multiplies the length with $\frac{1}{2}$. Since $L_0 = 1$, we get $L_n = \left(\frac{1}{2}\right)^n$ and hence $L_\infty = \lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$.

11.3.7: c) At every stage the length is multiplied by $\frac{4}{3}$. With initial length L_0, we thus get $L_\infty = \lim_{n\to\infty} \left(\frac{4}{3}\right)^n \cdot L_0 = \infty$.

d) Every time we apply the Koch curve procedure to one side, we create a new triangle whose sides are scaled down by a factor 3 and we create 4 sides for the next step. If our original triangle has side a, then at step n, we create $3 \cdot 4^{n-1}$ new triangles, with sides $\frac{a}{3^n}$. Since, by Pythagoras, the area of an equilateral triangle of sides b is equal to $\frac{b^2 \cdot \sqrt{3}}{4}$, we get

$$\text{area}(S) = \sum_{n=1}^{\infty} 3 \cdot 4^{n-1} \cdot \frac{a^2}{3^{2n}} \cdot \frac{\sqrt{3}}{4} + \frac{a^2\sqrt{3}}{4}$$

$$= \frac{a^2\sqrt{3}}{4} \left(\sum_{n=0}^{\infty} \frac{4^n}{9^n} \cdot \frac{1}{3} + 1 \right)$$

$$= \frac{a^2\sqrt{3}}{4} \cdot \left(\frac{1}{3} \cdot \frac{1}{1 - 4/9} + 1 \right)$$

$$= a^2 \frac{2\sqrt{3}}{5}.$$

e) ~~The snowflake consists of~~

Each side of the snowflake consists of 4 copies of itself, each scaled down by a factor 3. Hence the self-similarity dimension is $\frac{\ln 4}{\ln 3}$.