10.4.1

a) At a tangent bifurcation, we will have
\[
\begin{align*}
\text{re}^x &= x \\
\frac{d}{dx} \text{re}^x &= \frac{d}{dx} x
\end{align*}
\]
\[
\Rightarrow \begin{cases} 
\text{re}^x = x \\
\text{re}^x = 1
\end{cases}
\]

b) At a tangent bifurcation, we will have
\[
\begin{align*}
\text{re}^x &= x \\
\frac{d}{dx} \text{re}^x &= \frac{d}{dx} x
\end{align*}
\]
\[
\Rightarrow \begin{cases} 
\text{re}^x = x \\
\text{re}^x = 1
\end{cases}
\]

10.5.1

\[f(x) = rx \Rightarrow f'(x) = r.\]

Plugging into the definition of the Liapunov exponent, we get
\[
\lambda = \lim_{n \to \infty} \left( \frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)| \right) = \lim_{n \to \infty} \left( \frac{1}{n} \sum_{i=0}^{n-1} \ln |r| \right) = \ln |r|.
\]

11.3.1

a) The middle half Cantor set consists of 2 copies of itself, each scaled down by a factor 4. Hence its similarity dimension is \[\ln(2) = \frac{1}{\ln(4)}.\]

b) Removing the middle half of a line segment, multiplies the length with \[\frac{1}{2}.\] Since \[L_0 = 1\], we get \[L_n = \left(\frac{1}{2}\right)^n\] and hence \[L_\infty = \lim_{n \to \infty} \left(\frac{1}{2}\right)^n = 0.\]
11.3.7. c) At every stage the length is multiplied by $\frac{4}{3}$. With initial length $L_0$, we thus get $L_\infty = \lim_{n \to \infty} \left(\frac{4}{3}\right)^n L_0 = \infty$.

d) Every time we apply the Koch curve procedure to one side, we create a new triangle whose sides are scaled down by a factor 3 and we create 4 sides for the next step. If our original triangle has side $a$, then at step $n$, we create $3 \cdot 4^{n-1}$ new triangles, with sides $\frac{a}{3^n}$. Since, by Pythagoras, the area of an equilateral triangle of sides $b$ is equal to $\frac{b^2 \sqrt{3}}{4}$, we get

$$\text{area}(S) = \sum_{n=0}^{\infty} 3 \cdot 4^{n-1} \cdot \frac{a^2}{3^{3n}} \cdot \frac{\sqrt{3}}{4} + \frac{a^2 \sqrt{3}}{4}$$

$$= \frac{a^2 \sqrt{3}}{4} \left( \sum_{n=0}^{\infty} \frac{4^n}{9^n} \cdot \frac{1}{3} + 1 \right)$$

$$= \frac{a^2 \sqrt{3}}{4} \cdot \left( \frac{1}{1 - \frac{1}{4^{1/3}}} + 1 \right)$$

$$= \frac{a^2 \cdot 2 \sqrt{3}}{5}.$$

e) The snowflake consists of 4 copies of itself, each scaled down by a factor 3. Hence the self-similarity dimension is $\frac{\ln 4}{\ln 3}$. 