1. Calculate the character $\chi_{[3,2,1]}((123)(45)(6))$ of the symmetric group $S_6$.

2. Let $G$ be the group of order 21 given by generators $s$ and $t$ with relations $t^7 = s^3 = 1$ and $sts^{-1} = t^2$. Calculate $\chi(t)$ and $\chi(s)$ for all simple characters of $G$ (Hint: Consider representations induced from one-dimensional representations of a suitable subgroup). How does $CG$ decompose as a direct sum of simple matrix rings?

3. Let $G$ be a group all of whose characters are integers. Let $V$ be an irreducible representation of $G$ with $\dim V > 1$. Show that there exists a $g \in G$ for which $\chi_V(g) = 0$.

4. Let $I$ be a minimal ideal in an algebra $A$ satisfying $I^2 \neq 0$ ($I^2$ is the linear span of all elements of the form $ab$, with $a, b \in I$). Show that there exists an idempotent $p \in A$ such that $I = Ap$. (Hint: For given $b \in I$, consider the map $\rho_b : I \rightarrow I$, $a \in I \mapsto ab$).

5. Let $\epsilon(\pi)$ be the sign of the permutation $\pi$ and let $A_n$ be the subgroup of $S_n$ defined by $A_n = \{\pi \in S_n, \epsilon(\pi) = 1\}$.
   (a) Show that $\epsilon$ is a character of $S_n$, and, if $\chi$ is a character of $S_n$, then so is $\epsilon \chi$.
   (b) Assume that $\chi$ is a simple character of $S_n$ for which $\epsilon \chi = \chi$. Calculate $\chi(\pi)$ for $\pi \in A_n$.
   (c) Determine how the representation of $S_n$ corresponding to $\chi$ decomposes if it is restricted to $A_n$. What would be the dimensions of its subrepresentations?