MATH 202 2009: PRACTICE PROBLEMS

1. Calculate the character $\chi_{[3,2,1]}((123)(45)(6))$ of the symmetric group S_6.
2. Let G be the group of order 21 given by generators s and t with relations $t^7 = s^3 = 1$ and $sts^{-1} = t^2$. Calculate $\chi(t)$ and $\chi(s)$ for all simple characters of G ($Hint$: Consider representations induced from one-dimensional representations of a suitable subgroup). How does $\mathbf{C}G$ decompose as a direct sum of simple matrix rings?
3. Let G be a group all of whose characters are integers. Let V be an irreducible representation of G with $\dim V > 1$. Show that there exists a $g \in G$ for which $\chi_V(g) = 0$.
4. Let I be a minimal ideal in an algebra A satisfying $I^2 \neq 0$ (I^2 is the linear span of all elements of the form ab, with $a, b \in I$). Show that there exists an idempotent $p \in A$ such that $I = Ap$. ($Hint$: For given $b \in I$, consider the map $\rho_b : I \to I$, $a \in I \mapsto ab$).
5. Let $\varepsilon(\pi)$ be the sign of the permutation π and let A_n be the subgroup of S_n defined by $A_n = \{\pi \in S_n, \varepsilon(\pi) = 1\}$.
   (a) Show that ε is a character of S_n, and, if χ is a character of S_n, then so is $\varepsilon\chi$.
   (b) Assume that χ is a simple character of S_n for which $\varepsilon\chi = \chi$. Calculate $\chi(\pi)$ for $\pi \in A_n$.
   (c) Determine how the representation of S_n corresponding to χ decomposes if it is restricted to A_n. What would be the dimensions of its subrepresentations?