As before, whenever I refer to the book by Brian Hall, I am referring to the first edition.

1. (a) Problem 27 on page 62.
   (b) Show that $U(n)$ has a universal cover which is homeomorphic to $\mathbb{R} \times SU(n)$. (You may assume that $SU(n)$ is simply connected). Calculate the fundamental group of $U(n)$.

2. (a) Problem 10 on page 89.
   (b) Recall that we defined the universal cover $\hat{G}$ of an Lie group as the set
   
   $$\{ \gamma : [0, 1] \to G \text{ continuous, } \gamma(0) = I \}/\text{homotopy},$$

   i.e. homotopic paths get identified. Show that $\hat{G}$ is a group, with multiplication of paths defined as in part (a). (Ask me if this is too vague).

3. (a) Problem 13 on page 90
   (b) Show that any representation of $\hat{Sl}(n, \mathbb{R})$ is well-defined on the quotient
   
   $$\hat{Sl}(n, \mathbb{C}) / \pi_1(Sl(n, \mathbb{C})) \cong Sl(n, \mathbb{C}).$$

   *Hint*: You may use that $\hat{Sl}(n, \mathbb{R})$ has a Lie algebra which is isomorphic to $sl(n, \mathbb{R})$ such that any homomorphism $\Phi : \hat{Sl}(n, \mathbb{R}) \to Gl(m, \mathbb{C})$ induces a Lie algebra homomorphism $\psi : sl(n, \mathbb{R}) \to gl(m, \mathbb{C})$.
   (c) Show that $\hat{Sl}(n, \mathbb{R})$ can not be a matrix Lie group.