1. Let $f, g : [a, b] \to \mathbb{R}$ be two functions such that

$$|f(x) - f(y)| \leq |g(x) - g(y)| \quad \text{for all } x, y \in [a, b].$$

Assume $g$ is integrable. Show that $f$ is integrable.

2. Let $f[a, b] \to \mathbb{R}$ be a bounded function. Assume that there exist two partitions $P_1$ and $P_2$ such that

$$U(f, P_1) \leq L(f, P_2).$$

Prove that $f$ is a constant function.