You should proceed in two steps, as it was done in class on Monday. First show that the sequence converges to a limit, and then calculate the limit using the trick that a subsequence converges to the same limit as the original sequence.

One of the difficulties is that the sequence \((x_n)\) may be increasing, or decreasing, depending on the value of \(x_1\). The following should be helpful:

Let \(\ell\) be the positive solution of the equation

\[ x^2 - x - c = 0. \]

You can easily write down a formula for \(\ell\). Show that if \(\ell < x_n\), then

\[ \ell < x_{n+1} < x_n. \]

*Hint*: It may be convenient to use \(\ell^2 = \ell + c\), or, equivalently \(\ell = \sqrt{\ell + c}\). Also, draw the graph of the function \(f(x) = x^2 - x - c\) to show that \(f(x) > 0\) if \(x > \ell\).

Similarly, if \(0 < x_n < \ell\), show that

\[ x_n < x_{n+1} < \ell. \]