Math 18: Lectures ABC **Midterm Exam 2 v. 1** **Name:** ______________________

February 28, 2018 **(Total Points: *30*)** **PID:** ______________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
5. **Read each question carefully and answer each question completely.**
6. **Write your solutions clearly in the spaces provided.**
7. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the matrix $A = \begin{bmatrix} 2 & 3 & 0 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$.

(a) Calculate the third column of A^{-1}.

(b) Let $B = \frac{1}{2}AA^{\top}$. Calculate $\det(B^{-1})$.

(9 points) 2.

The matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 2 & 4 & 1 & 9 & 3 \\ 1 & 2 & -1 & -3 & 3 \end{bmatrix}$ has reduced row-echelon form $\begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 0 & 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathrm{Nul}(A)$.

(b) Find a basis for $\mathrm{Col}(A)$.

(c) Find a basis for $\mathrm{Col}(A^\top)$.

(8 points) 3. In each of the following examples, a vector space V is given, along with a subset S. Determine whether S is a subspace or not. In each case, explain why it is or is not a subspace.

(a) $V = M_{4\times 5}$ is the space of 4×5 matrices, and S is the set of 4×5 matrices with rank 1.

(b) $V = M_{1\times 3}$ is the space of 3-dimensional row vectors, and $S = \operatorname{span}\{[1, 1, 1], [-2, 2, 3]\}$.

(c) $V = \mathbb{R}^2$, and $S = \left\{ \begin{bmatrix} t \\ 3t \end{bmatrix} : 0 \le t \le 2 \right\}$.

(d) $V = \mathbb{P}_2$ is the space of polynomials of degree ≤ 2, and S is the subset of polynomials p in V for which $p(1) = 0$.

(6 points) 4. Let $H = \left\{ \begin{bmatrix} s+2t \\ 2s-t \\ t \end{bmatrix} : s,t \in \mathbb{R} \right\}$. H is a subspace of $\mathbb{R}^3$, with basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$.

(a) The vector $\mathbf{v} = \begin{bmatrix} 2 \\ 9 \\ -1 \end{bmatrix}$ is in H. Find its coordinate vector $[\mathbf{v}]_{\mathcal{B}}$.

(b) Let $\mathbf{u}$ be the sum of the two basis vectors in $\mathcal{B}$. Is $\{\mathbf{u}, \mathbf{v}\}$ a basis for H? Explain why or why not.