1. Calculate the multiplicity of the $S_{20}$ module $V_{[6,5,4,3,2]}$ in the representation induced from the $S_{14} \times S_6$ module $V_{[5,4,3,2]} \otimes V_{[3,2,1]}$.

2. Let $\lambda$ be a Young diagram with $\leq k$ rows, and let $H_\lambda = \prod_{i=1}^k H_{\lambda_i}$, where $H_n = \sum_{\alpha \vdash n} \alpha! x_\alpha$ in the variables $x_1, \ldots, x_k$. Show that $(H_\lambda)_{\lambda \leq k}$ rows forms a basis of the symmetric functions. Moreover, show that any monomial symmetric function and any Schur function is a linear combination with integer coefficients of the $H_\lambda$s.

3. Let $f = xy^2 - x$, $f_1 = xy + 1$ and $f_2 = y^2 - 1$.
   (a) Do division with remainder, first dividing $f$ by $f_1$ and then by $f_2$.
   (b) Do the same with $f_1$ and $f_2$ interchanged. Compare.

4. (a) Let $f$ be an irreducible polynomial over $k$ in one variable $x$ (i.e. $f$ can not be written as $f = gh$ with both $g$ and $h$ polynomials of degree $\geq 1$). Show that if $\bar{f} \not\in \langle f \rangle$, the ideal generated by $f$, then $\langle f, \bar{f} \rangle = k[x]$.
   (b) Let now $f \in k[x]$ be arbitrary, and let $\langle f \rangle \subset I_1 \subset I_2 \subset \ldots$ be a sequence of ideals, with strict inequalities. Show that this sequence has to be finite.

5. Let $A = \bigoplus_{\lambda} A_\lambda$ be a semisimple algebra (say over the complex numbers $\mathbb{C}$), with $A_\lambda$ being isomorphic to the algebra of $d_\lambda \times d_\lambda$ matrices with $d_\lambda \in \mathbb{N}$. Let $e \in A$ be an idempotent, and let $r_\lambda = Tr_{V_\lambda}(e)$, where $V_\lambda$ is a simple $A_\lambda$-module. The vector $\vec{r} = (r_\lambda)$ is called the rank vector of $e$.
   (a) Let $V$ be an $A$-module, with multiplicity vector $\vec{m} = (m_\lambda)$; i.e. $m_\lambda$ is the number of modules $V_i$ isomorphic to $V_\lambda$ in the decomposition of $V = \oplus_i V_i$ of simple $A$-modules. What is the rank of the projection representing $e$ on $V$, i.e. the dimension of the subspace $eV$?
   (b) Let $e \in \mathbb{C}S_4$ with rank vector $\vec{r} = (r_\lambda)$, where $r_\lambda = 0$ if $\lambda$ has one or two rows, and $r_\lambda = 1$ if $\lambda$ has more than two rows. Calculate the rank of the projection via which $e$ acts on $V^\otimes 4$, with dim $V = 3$. 