EXERCISES MATH 202C - First Assignment

1. Let $A_\lambda(x_1, \ ..., \ x_N) = \det(x_j^{\lambda_i+N-i})$ and define $s_\lambda = A_\lambda/A_0$.
   (a) Show that $A_\lambda(q^{N-1}, q^{N-2}, \ ..., \ q, 1) = A_0(q^{\lambda_1+N-1}, q^{\lambda_2+N-2}, \ ..., \ q^{\lambda_N})$.
   (b) Calculate $s_\lambda(1, 1, \ ..., \ 1)$ ($Hint$ : Recall that $A_0(x_1, x_2, \ ..., \ x_N)$ can be written as product, and use (a)).
   (c) Calculate the dimension of the subspace $e_\lambda V^{\otimes n}$ of $V^{\otimes n}$, where $e_\lambda$ is a minimal idempotent of $\mathbf{C}S_n$.

2. Let $d_\lambda$ be the dimension of the simple $S_n$-module $S^\lambda$. Calculate $\sum_{\mu \supset \lambda} d_\mu$, where the summation goes over all Young diagrams obtained by adding one box to $\lambda$. $Hint$ : For my favorite solution, consider induced representations.

3. (need not be turned in) Look at those problems of the previous final with which you had difficulties. If you still have difficulties, discuss with me or the TA.