HW 5 PROBLEMS

From Rudin Chapter 3 solve: 1,3,16,17,20,21,23.
Also solve the following problems:

**Problem 1.** Assume that \( \{x_n\} \) is a Cauchy sequence in the metric space \( X \) and that its range is finite. Prove that there exists a constant \( c \in X \) and \( N \) such that \( x_n = C, \forall n \geq N \).

**Problem 2.** (Squeeze Lemma) If \( \{a_n\}, \{b_n\}, \{c_n\} \) be real-valued sequences with the property that
\[
a_n \leq b_n \leq c_n.
\]
If \( \lim_{n \to \infty} a_n = l \) and \( \lim_{n \to \infty} c_n = l \), then prove that \( \lim_{n \to \infty} b_n = l \).

**Problem 3.** Prove that if the real-valued sequence \( \{a_n\} \) converges to 0, and the sequence \( \{b_n\} \) is bounded, then \( \lim_{n \to 0} a_n b_n = 0 \).

**Problem 4.** Prove that if the real-valued sequence is such that \( \lim_{n \to \infty} |a_n| = 0 \), then \( \lim_{n \to \infty} a_n = 0 \).