HW 5 PROBLEMS

From Rudin Chapter 3 solve: 1,3,16,17,20,21,23.

Also solve the following problems:

Problem 1. Assume that $\{x_n\}$ is a Cauchy sequence in the metric space X and that its range is finite. Prove that there exists a constant $c \in X$ and N such that $x_n = C, \forall n \geq N$.

Problem 2. (Squeeze Lemma) If $\{a_n\}, \{b_n\}, \{c_n\}$ be real-valued sequences with the property that

$$a_n \leq b_n \leq c_n.$$

If $\lim_{n\to\infty} a_n = l$ and $\lim_{n\to\infty} c_n = l$, then prove that $\lim_{n\to\infty} b_n = l$.

Problem 3. Prove that if the real-valued sequence $\{a_n\}$ converges to 0, and the sequence $\{b_n\}$ is bounded, then $\lim_{n\to\infty} a_n b_n = 0$.

Problem 4. Prove that if the real-valued sequence is such that $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.