HW 7 PROBLEMS

From Rudin 21 from Chapter 7, 1,3 from Chapter 8. Also solve the following problems:

Problem 1. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence R. If the power series converges at $x = R$, prove that the series converges uniformly on $[0, R]$.

Problem 2. We have shown that

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| \leq 1.$$

Use this formula together with other results we establish to compute the exact value of the following series:

$$\sum_{n=1}^{\infty} \frac{n}{2^n}, \quad \sum_{n=1}^{\infty} \frac{n^2}{2^n}.$$

Problem 3. Assume $f : \mathbb{R} \to \mathbb{R}$ is infinitely many times differentiable. We have proved in Math 140 that if

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

then there exists c between x and x_0 with the property that

$$f(x) = P_n(x) + \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}.$$

Assume that for any $R > 0$, there exists M with the property $|f^{(n)}(x)| \leq M, \forall |x| \leq R$. Prove that the Taylor series

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

converges uniformly to f on any compact subset of $\mathbb{R}$.