Practice Midterm 1

Problem 1. Prove that for any \( n \in \mathbb{N} \) and any \( a_1, ..., a_n \in \mathbb{R} \), the following holds true:
\[
|a_1 + a_2 + ... + a_n| \leq |a_1| + ... + |a_n|.
\]
Hint: for a rigorous proof you may want to use induction.

Problem 2. Prove that \( 6^{\frac{1}{3}} \) is not a rational number.

Problem 3. a) Write down the definition of a convergent sequence.
   b) Prove that \( \lim_{n \to \infty} \frac{1}{n^2} = 0 \); Note: either you do a direct proof or else you prove any result that you are using.
   c) Prove that the sequence \( 1 + (-1)^n \) does not converge.
   d) Prove that the limit of a convergent sequence is unique.

Problem 4. Let \( (a_n) \) be a sequence in \( \mathbb{R} \). Prove that \( (a_n) \) converges to 0 if and only if \( (|a_n|) \) converges to 0.

Problem 5. Suppose that \( S \subset \mathbb{R} \) is a non-void bounded set. If \( \inf S = \sup S \), prove that \( S \) contains only one element.