Practice Midterm 1

Problem 1. Prove that for any $n \in \mathbb{N}$ and any $a_1, .., a_n \in \mathbb{R}$, the following holds true:

$$|a_1 + a_2 + ... + a_n| \leq |a_1| + ... + |a_n|.$$

Hint: for a rigorous proof you may want to use induction.

Problem 2. Prove that $6^{\frac{1}{3}}$ is not a rational number.

Problem 3. a) Write down the definition of a convergent sequence.

b) Prove that $\lim \frac{1}{n^2} = 0$; Note: either you do a direct proof or else you prove any result that you are using.

c) Prove that the sequence $1 + (-1)^n$ does not converge.

d) Prove that the limit of a convergent sequence is unique.

Problem 4. Let (a_n) be a sequence in $\mathbb{R}$. Prove that (a_n) converges to 0 if and only if $(|a_n|)$ converges to 0.

Problem 5. Suppose that $S \subset \mathbb{R}$ is a non-void bounded set. If $infS = supS$, prove that S contains only one element.