HW 6

Problem 1: 30.2(a,c),30.3(a,c),30.7;
Problem 2: 31.2,31.6;
Problem 3. The purpose of this problem is to establish that

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}, \quad \forall x \in (-1,1], \tag{1}$$

using the Taylor series theory, but without using the power series theory as it was done in Chapter 4.26, Example 1.

a) Following the argument in Example 2 in Chapter 5.31, where the equality (1) is shown for $x = 1$, prove that the equality (1) holds true for $x \in [0,1]$.

b) Following the sane strategy as in a), prove the equality (1) holds true for $x \in [-\frac{1}{2}, 0]$ as well; Hint: prove that if $y \in (x,0)$ and $-\frac{1}{2} \leq x \leq 0$, then $|\frac{x}{1+y}| \leq 1$.

c) Prove that (1) holds true for $x \in (-1, -\frac{1}{2})$.

Hint: to show that the equality (1) holds true for $x \in (-1, -\frac{1}{2})$, one cannot rely anymore on Taylor's theorem 31.3, but instead use Corollary 31.6. In fact, you can use an argument that is similar to the one in the binomial series theorem 31.7 in Chapter 5.31.