Name: ______________________________ **PID:** ________________

TA: ______________________ **Sec. No:** _____ **Sec. Time:** _____

Math 10C (Klep)
Final Exam
March 17, 2008

Turn off and put away your cell phone.
You may use one sheet of notes, but no calculators, books or other assistance.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	7	
3	8	
4	9	
5	4	
6	6	
7	8	
8	12	
Σ	60	

(1) (6 pts) The function $z = f(x, y)$ is given by its values in the following table:

	x		
y	0	1	2
0	0	2	4
1	3	5	7
2	6	8	10

(a) Why is this data consistent with that coming from a linear function?

Be cause the difference between a box and the box to its left is 2 and the difference between a box and the box above is 3

(b) Find a linear function whose values are those in the table. Your answer should be of the form $z = m\,x + n\,y + c$.

$z = 2x + 3y$

(2) (7 pts) Let $\vec{v} = -3\vec{i} + \vec{k}$, $\vec{u} = 2\vec{j} - \vec{k}$ and $\vec{w} = \vec{i} - 2\vec{j}$.

Consider the following expressions:

(i) $\vec{v} - (\vec{u} \cdot \vec{w})$,

(ii) $\vec{v} \cdot (\vec{u} \times \vec{w})$,

(iii) $\vec{v} \times (\vec{w} \cdot \vec{u})$.

(a) Which of the expressions make sense? Explain!

(i) Doesn't make sense $u \cdot w$ is a scalar and subtraction of a scalar from a vector is not defined

(ii) makes sense. $u \times w$ is a vector

(iii) $v \times (w \cdot u)$ doesn't make sence since $w \cdot u$ is a scalar and cross product operates on vectors

(b) Compute all meaningful expressions.

$v \cdot (u \times w)$

$$u \times w = \begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ 1 & -2 & 0 \end{vmatrix} = -j - 2k - 2i$$

$$v \cdot (u \times w) = (-3)(-2) + (-2)(1) = 4$$

(3) (8 pts) Suppose

$$P_2(x,y) = -1 + x - 3y - \frac{1}{2}x^2 + 3y^2$$

is the second degree Taylor polynomial for the function $f(x,y)$ at the point $(0,0)$.

(a) Find $f(0,0)$, $f_x(0,0)$ and $f_y(0,0)$.

$f(0,0) = -1$

$f_x(0,0) = 1$

$f_y(0,0) = -3$

(b) Determine the tangent plane to the graph of $f(x,y)$ at the point $(0,0,-1)$.

$z = -1 + x - 3y$

(c) Find $f_{xx}(0,0)$, $f_{xy}(0,0)$, $f_{yy}(0,0)$.

$f_{xx}(0,0) = -1$

$f_{xy}(0,0) = 0$

$f_{yy}(0,0) = 6$

(4) (9 pts) A closed rectangular box with square bottom has surface area 54 cm^2. In this question we want to minimize the sum of the box's width, length and height.

(a) Find a constrained optimization problem over two variables whose solution gives the minimum sum of the width, length and height of the box.

$$l = w$$

$$2lw + 2wh + 2lh = 54$$

constra. –

$$g(l,h) = l^2 + 2lh = 27$$

objective function

$$f(l,h) = 2l + h$$

(b) Use Lagrange multipliers to solve the optimization problem.

$$\text{grad } g = (2l + 2h)\vec{i} + 2l\vec{j}$$

$$\text{grad } f = 2\vec{i} + \vec{j}$$

$$\text{grad } f = \lambda \text{ grad } g$$

$$\lambda(2l + 2h) = 2$$

$$\lambda(2l) = 1$$

so $2\lambda h = 1$

$$27 = \left(\frac{1}{2\lambda}\right)^2 + 2\left(\frac{1}{2\lambda}\right)^2$$

$$27 \cdot 4\lambda^2 = 3$$

$$\lambda = \pm\frac{1}{6}$$

$l = 3 \quad w = 3 \quad h = 3$

min sum $= 9$

(5) (4 pts) To each of the following functions assign the corresponding contour diagram. No justification required.

$z = \sqrt{\frac{1}{4}x^2 + y^2}$	$z = 2x^2 - 1$	$z = 3x^2 - 2y^2$	$z = \frac{1}{4}x^2 + y^2$
C	D	A	B

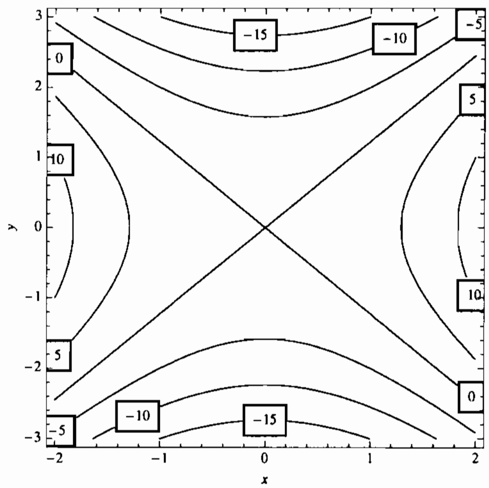


Diagram A

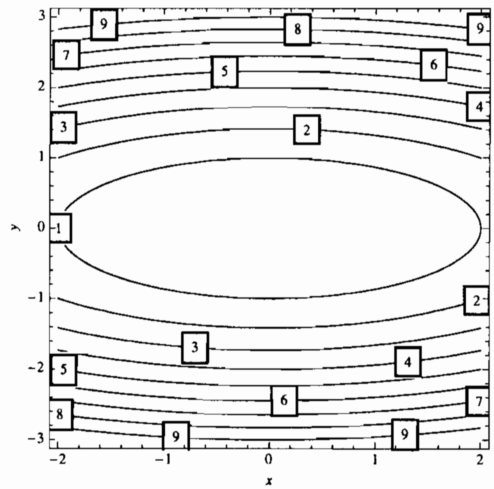


Diagram B

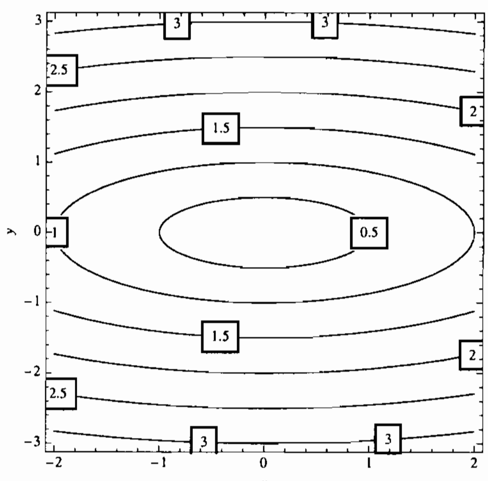


Diagram C

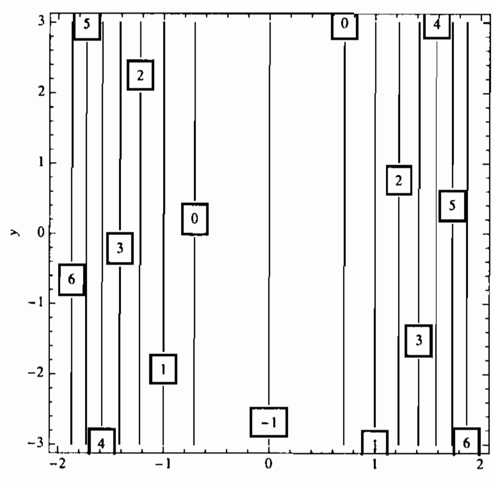
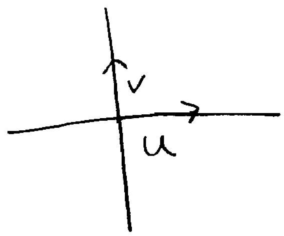


Diagram D

(6) (6 pts) A surfer in the water near UCSD is being attacked by a shark. The surfer is desperately trying to swim straight east toward the shore, while the shark is trying to drag him north. The shark's top speed is 5 ft/s and the surfer's is $5\sqrt{3}$ ft/s.

(a) Let $\vec{u}$ be the velocity vector of the surfer and $\vec{v}$ be the velocity vector of the shark. Find $\vec{u}$ and $\vec{v}$ in component form.



$u = 5\sqrt{3}\,\vec{i}$

$v = 5\,\vec{j}$

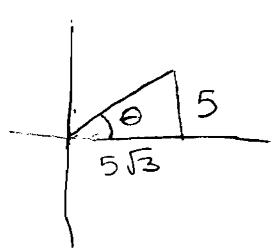
(b) Give the vector $\vec{w}$ representing the actual movement of the surfer with the shark clinging to his leg. How fast are they moving?

$w = u + v$

$= 5\sqrt{3}\,\vec{i} + 5\,\vec{j}$

speed: $\|w\| = \sqrt{25 \cdot 3 \cdot 25} = 10$ ft/s

(c) By what angle are they moving off the surfer's ideal course of straight east?



$\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) = 30^\circ$

(7) (8 pts) Let $f(x, y) = e^{x-y^2}$.

(a) Compute the gradient of $f(x, y)$ at the point $(4, 2)$.

$F_x(x, y) = e^{x-y^2}$

$F_y(x, y) = -2y\, e^{x-y}$

$\text{grad } F = \bar{i} - 4\bar{j}$

(b) What is the maximum rate of change of $f(x, y)$ at the point $(4, 2)$? What is the direction of the maximum rate of change of $f(x, y)$ at the point $(4, 2)$?

max rate of change $= \| i - 4j \| = \sqrt{4^2 + 1^2} = \sqrt{17}$

direction $= \bar{i} - 4\bar{j}$

(c) Find the rate of change of $f(x, y)$ in the direction of $\vec{v} = 4\vec{i} - 3\vec{j}$ at the point $(4, 2)$. Note: $\vec{v}$ is not a unit vector.

$$\frac{V}{\|V\|} = \frac{4i - 3j}{5}$$

rate of change $= (\bar{i} - 4\bar{j}) \cdot (\tfrac{4}{5}i - \tfrac{3}{5}j)$

$= \tfrac{4}{5} + \tfrac{12}{5} = \tfrac{16}{5}$

(8) (12 pts) Let $f(x, y) = x^4 + y^2 + 2xy$.

(a) Find f_x, f_y, f_{xx}, f_{xy} and f_{yy}.

$F_x = 4x^3 + 2y$

$F_y = 2x + 2y$

$f_{xx} = 12x^2$

$F_{xy} = 2$

$F_{yy} = 2$

(b) Find all critical points of $f(x, y)$.

$0 = F_x(x,y) = 4x^3 + 2y$

$0 = F_y(x,y) = 2x + 2y$

$x = -y$

$0 = 4x^3 - 2x = 2x(2x^2 - 1) = 2x(\sqrt{2}x + 1)(\sqrt{2}x - 1)$

$(0,0)$, $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

(c) Characterize the critical points of $f(x, y)$ as local minima, local maxima or neither.

$(0,0)$ $D = 0 - 4 < 0$

$\Rightarrow (0,0)$ is a saddle point

$(1,-1)$ $D = \frac{12}{2}\ 2 - 4 > 0$

$f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \cdot 12 > 0$

so $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ is a loc min

$(-1,1)$ $D = \frac{12}{2} \cdot 2 - 4 > 0$

$f_{xx}\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = 12 > 0$

so $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ is a loc min