

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 10C (Klep)
Midterm Exam 2
February 27, 2008

Turn off and put away your cell phone.

You may use one page of notes, but no calculators, books or other assistance.

Read each question carefully, and answer each question completely.

Show all of your work; no credit will be given for unsupported answers.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	8	
2	10	
3	10	
4	12	
Σ	40	

1. Let $\vec{u} = (6, 1, -2)$ and $\vec{v} = 3\vec{i} - 2\vec{k}$.

(a) (5 pts) Find a vector orthogonal to both \vec{u} and \vec{v} .

$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 1 & -2 \\ 3 & 0 & -2 \end{vmatrix} \\ &= -2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}} + 12\hat{\mathbf{j}} = -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}} \end{aligned}$$

(b) (3 pts) What are two unit vectors orthogonal to both \vec{u} and \vec{v} ?

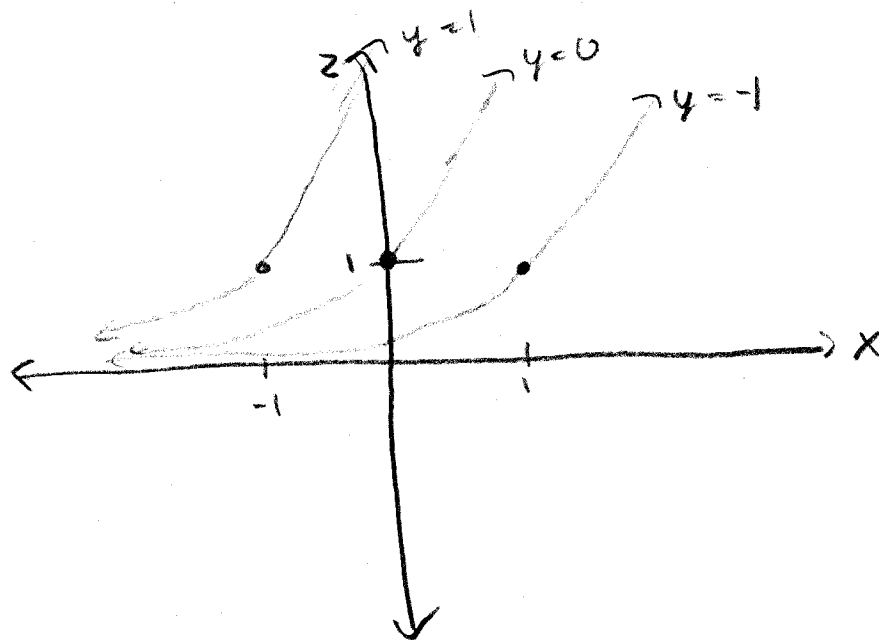
$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{4 + 36 + 9} = 7$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = -\frac{2}{7}\hat{\mathbf{i}} + \frac{6}{7}\hat{\mathbf{j}} - \frac{3}{7}\hat{\mathbf{k}}$$

$$\text{also } \frac{2}{7}\hat{\mathbf{i}} - \frac{6}{7}\hat{\mathbf{j}} + \frac{3}{7}\hat{\mathbf{k}}$$

2. Consider the function $z = f(x, y) = e^{x-y}$.

(a) (5 pts) Plot the vertical cross-sections when y is fixed to $-1, 0$ and 1 .

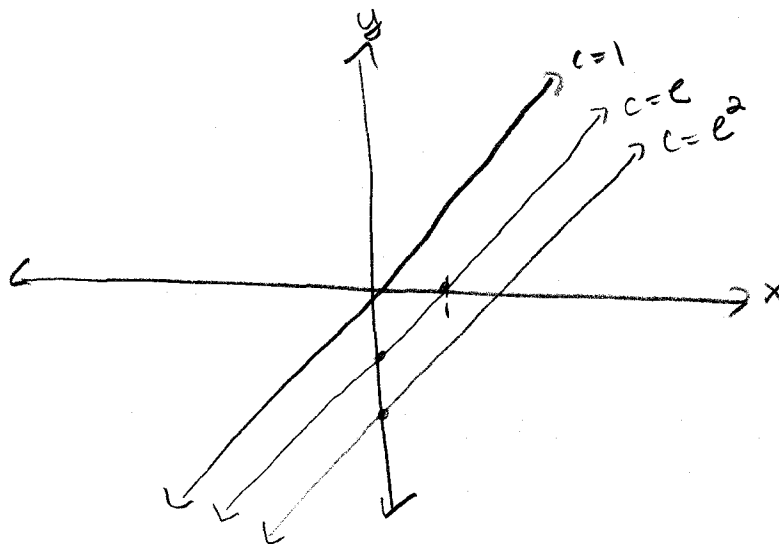


(b) (5 pts) Write down the equations for the contours at heights $1, e,$ and $e^2,$ and draw a contour diagram with these three contours clearly labeled.

$$1 = e^{x-y} \text{ iff } 0 = x - y$$

$$e = e^{x-y} \text{ iff } 1 = x - y$$

$$e^2 = e^{x-y} \text{ iff } 2 = x - y$$



3. Let

$$f(x, y) = \frac{\sin(x + 2y)}{x^2 + 1}$$

- (a) (6 pts) Find the equation of the tangent plane to the graph of the function $f(x, y)$ at the point $(0, \frac{\pi}{4}, 1)$.

$$F_x = \frac{\cos(x+2y)(x^2+1) - 2x \sin(x+2y)}{(x^2+1)^2}$$

$$F_x(0, \pi/4) = 0$$

$$F_y = \frac{2 \cos(x+2y)}{x^2+1}$$

$$F_y(0, \pi/4) = 0$$

$$Z = F(0, \pi/4) = 1$$

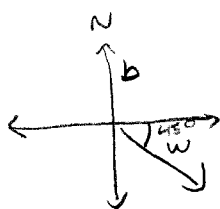
- (b) (4 pts) Use this equation to estimate $f(0.1, 1)$.

$$F(x, y) = 1$$

$$\Rightarrow F(.1, 1) \approx 1$$

4. A yacht is heading due north at $5 \cdot (1 + \sqrt{3})$ km/hr (relative to the water). The current is moving toward the southeast at $5 \cdot \sqrt{2}$ km/hr.

(a) (4 pts) Let \vec{v} be the velocity vector of the current and let \vec{u} be the velocity vector of the yacht. Find \vec{v} and \vec{u} in component form.



$$\vec{u} = 5(1 + \sqrt{3})\hat{j}$$

$$\vec{v} = \omega_1 \hat{i} - \omega_1 \hat{j}$$

$$5\sqrt{2} = \|\vec{v}\| = \sqrt{\omega_1^2 + \omega_1^2} = \sqrt{2} \omega_1$$

$$\Rightarrow \omega_1 = 5$$

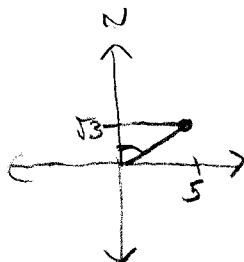
$$\vec{v} = 5\hat{i} - 5\hat{j}$$

(b) (4 pts) Give the vector $\vec{w} = (w_1, w_2)$ representing the actual movement of the yacht. How fast is the yacht going, relative to the ground?

$$\vec{w} = \vec{u} + \vec{v} = 5(1 + \sqrt{3})\hat{j} + 5\hat{i} - 5\hat{j} = (5, 5\sqrt{3})$$

$$\text{speed} = \|\vec{w}\| = \sqrt{25 + 25 \cdot 3} = 10$$

(c) (4 pts) By what angle does the current push the yacht off of its due north course?



$$\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$