PRACTICE MIDTERM 1

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Math 100A, Lecture B Fall 2018

(1) If \( p \neq 2 \) is a prime number, show that \( p \equiv 1 \) or \( 3 \) mod 4.

(2) For an element \( a \) in \( \mathbb{Z}_n \), can there both exist \( a^{-1} \) in \( \mathbb{Z}_n \) and \( c \) in \( \mathbb{Z}_n \) such that \( ac = 0 \)? Why or why not?

(3) For any integer \( a \), prove that 3 divides \( a \) if and only if 3 divides the sum of the digits of \( a \).

(4) Factor \((12345)(67)(1357)(163)\) into disjoint cycles.

(5) Using cycle notation, list all elements \( \sigma \in S_4 \) such that \( \sigma^{-1} = \sigma \).

(6a) Give an example of a group that is not finite.

(6b) Give an example of a group of order at least 4 that is finite. Compute the orders of 4 distinct elements.

(7) If \( G \) is a group under the operation \( \ast \), define the operation \( \star \) on \( G \) by \( g \ast h = g^{-1} \ast h^{-1} \). Is \( G \) a group under \( \star \)? If so, prove it. If not, describe why it fails.

(8) Prove that if \( H \) is a subgroup of \( G \), and \( K \) is a subgroup of \( H \), then \( H \) is a subgroup of \( G \).

(9a) Describe all subgroups of \( S_4 \).

(9b) Which subgroups of \( S_4 \) are conjugate to one another? Which subgroups of \( S_4 \) are conjugate to themselves? Which are abelian?