Example: \( f(x) = \frac{1}{x-1} \). Determine the behavior as \( x \) approaches \( \pm \infty \) and as \( x \) approaches 1.

\[
\frac{1}{x} \to 0 \text{ as } x \to \pm \infty \text{ therefore } \frac{1}{x-1} \to 0 \text{ as } x \to \pm \infty
\]

\[
\frac{1}{x-1} \to \infty \text{ as } x \to +1 \text{ (from the right i.e. } x>1) \]

\[
\frac{1}{x-1} \to -\infty \text{ as } x \to +1 \text{ (from the left i.e. } x<1) \]

Horizontal asymptote at zero.

Vertical asymptote at \( x=1 \).
Ex: What is the equation of a circle with radius 3 and centered at (1,3)?

A circle is the set of points that are a fixed-distance (radius) from a center point.

All points \((x,y)\) that are a distance of 3 from the point \((1,3)\),

distance \((1,3), (x,y)\) = \(\sqrt{(x-1)^2 + (y-3)^2} = 3\)

\[
\Rightarrow (x-1)^2 + (y-3)^2 = 9
\]

Ex: How many zeros does this polynomial have?

Three zeros
Ex: How many zeros does this polynomial have?

(None.)

Ex: Find the largest interval over which the function \( f(x) = x^2 + 2x + 2 \) is an increasing function.

- We need to find the minimum value of \( f(x) \) and what \( x \) obtains that value.
- Completing the square helps us find those values.
\[ f(x) = (x^2 + 2x) + 2 = ((x+1)^2 - (0)^2) + 2 \]
\[ = (x+1)^2 + 1 \]

- Since \((x+1)^2 \geq 0\), the minimum value occurs when \(x = -1\), then 
  \((x+1)^2 = 0\) and \(f(-1) = (-1+1)^2 + 1 = 1\). So the vertex is at the point \((-1, 1)\), and everything to the right of the vertex is the interval over which \(f(x)\) is increasing, i.e. \([-1, \infty]\).

**Ex:** \[|x-1| + |x+2| = 3\]
Find all \(x\) that satisfy the equation.
- \(|x-1| = 0\) when \(x = 1\)
- \(|x+2| = 0\) when \(x = -2\)
For any $x \in (-\infty, -2)$, $(x - 1) < 0$ and $(x + 2) < 0$.

$\Rightarrow (x - 1 + 1|x + 2|) = 3$ becomes $-(x - 1) - (x + 2) = 3$

$\Rightarrow -2x - 1 = 3 \Rightarrow -2x = 4 \Rightarrow x = -2$

Notice $x = -2$ is where $1|x + 2| = 0$ and plugging $x = -2$ in $+ 0$

$|x - 1| + |x + 2| = 3$ yields $1 - 2 - 1 + |1 - 2 + 2| = 3$

$\Rightarrow 3 = 3$, so $x = -2$ is a solution!

Case 2: For any $x \in (-2, 1)$, $(x - 1) < 0$ and $(x + 2) > 0$.

$\Rightarrow |x - 1| + |x + 2| = 3$ becomes $-(x - 1) + (x + 2) = 3$

$\Rightarrow (x + x) + (1 + 2) = 3 \Rightarrow 3 = 3$!

This means for any $x \in (-2, 1)$ the equation $|x - 1| + |x + 2| = 3$.

Case 3: For any $x \in (1, \infty)$, $(x - 1) > 0$ and $(x + 2) > 0$.

$\Rightarrow |x - 1| + |x + 2| = 3$ becomes $x - 1 + x + 2 = 3$

$\Rightarrow 2x + 1 = 3 \Rightarrow 2x = 2 \Rightarrow x = 1$. Noting $x = 1$ is where $|x - 1| = 0$ we conclude $x = 1$ is a solution.
Therefore, all the possible solutions make up the interval \([-2, 1]\).

We can explain this graphically.

\[|x-1| + |x+2| = 3 \Rightarrow |x-1| = 3 - |x+2|\]

Let's graph \(f(x) = |x-1|\) and \(g(x) = 3 - |x+2|\).

\[\text{combined}\]