What values of \( x \) satisfy \( |x+1| + |x-3| = 6 \)?

We can split this into 3 cases by looking at where the absolute values equal zero.

\[ |x+1| = 0 \quad \text{when} \quad x = -1 \]
\[ |x-3| = 0 \quad \text{when} \quad x = 3 \]

Let's plot these on the real line.

\((-\infty, -1)\) \quad (-1, 3) \quad (3, \infty)\)

Now let's examine each interval separately. Remember we're looking for values of \( x \) within each interval that satisfy the equation.
Case 1: If \( x \) is in \((-\infty, -1)\), then
\[x+1<0 \text{ therefore } |x+1| = -(x+1),\]
and \( x-3<0 \text{ therefore } |x-3| = -(x-3)\).
The equation becomes
\[-(x+1) - (x-3) = 6,\]
\[-2x + 2 = 6,\]
\[-2x = 4,\]
\[x = -2.\]

Case 2: If \( x \) is in \((-1, 3)\), then
\[x+1>0 \text{ therefore } |x+1| = x+1,\]
and \( x-3<0 \text{ therefore } |x-3| = -(x-3)\).
The equation becomes,
\[x+1 - (x-3) = 6,\]
\[4 = 6 \text{ False!}\]
This means there are NO solutions in \((-1, 3)\).
Case 3: If $x$ is in $(3, \infty)$, then $x+1 > 0$ therefore $|x+1| = x+1$, and $x-3 > 0$ therefore $|x-3| = x-3$.

The equation becomes

$x + 1 + x - 3 = 6$

$2x - 2 = 6$

$2x = 8$

$x = 4$

After checking all cases we can conclude that $|x+1| + |x-3| = 6$ is satisfied by $x = -2$ and $x = 4$. 
1. Cartesian Plane
2. Graphs
3. Distance
4. Circumference/Perimeter

Cartesian Plane

Take two copies of the real line. Flip one vertically, so that the positive direction points up. Finally, lay one copy on top of the other, so that they intersect at zero.
Rectangular Coordinates

\((X, Y)\): the point with value \(X\) on the \(x\)-axis and value \(Y\) on the \(y\)-axis. \((0, 0)\) is called the "origin".
Graph of an Equation

Def: The graph of an equation with two variables (x and y) is the set of points in the corresponding coordinate plane satisfied by the equation.

Ex: \( y = x \)

\[
\begin{array}{c|c}
X & Y \\
1 & 1 \\
3 & 3 \\
-2 & -2 \\
-4 & -4 \\
\end{array}
\]

(should be a straight line.)
**Example:** $y = x^2$ (parabola)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>-3</td>
<td>9</td>
</tr>
</tbody>
</table>

**Example:** $x^2 + y^2 = 1$ (unit circle)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>
Euclidean Distance

How far is one point from another point?

**EX:** Consider \((0,0)\) and \((1,1)\)

\[
 \begin{align*}
  a &= 1 \\
  b &= 1 \\
  c &= ? \\
  1^2 + 1^2 &= c^2 \\
  2 &= c^2 \\
  \Rightarrow c &= \sqrt{2}
\end{align*}
\]
Def: The Euclidean distance between \((X_1, Y_1)\) and \((X_2, Y_2)\) is given by
\[
\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}
\]

Ex: \((0, 0)\) and \((1, 1)\)
\[
\sqrt{(1 - 0)^2 + (1 - 0)^2} = \sqrt{2}
\]

Ex: \((-4, 1)\) and \((2, 3)\)
\[
\sqrt{(2 - (-4))^2 + (3 - 1)^2} = \sqrt{36 + 4} = \sqrt{40}
\]
(order doesn't matter)
\[
\sqrt{(-4 - 2)^2 + (1 - 3)^2} = \sqrt{36 + 4} = \sqrt{40}
\]

Ex: \((2, -3)\) and \((2, -3)\)
\[
\sqrt{(2 - 2)^2 + (-3 - (-3))^2} = 0
\]
**Perimeter / Circumference**

**Def:** The perimeter of a polygon (i.e. square, triangle, etc) is the length of the path that surrounds the polygon.

**Def:** The circumference of a region is the length of the curve that surrounds the region.

**Ex:** Determine the perimeter of the following triangle.

![Diagram of a triangle with vertices at (1,1), (2,1), and (1,2)]
\((-1, -1)\) to \((2, 1)\)

\[
\sqrt{(2 - (-1))^2 + (1 - (-1))^2} = \sqrt{9 + 4} = \sqrt{13}
\]

\((-1, -1)\) to \((1, -2)\)

\[
\sqrt{(1 - (-1))^2 + (-2 - (-1))^2} = \sqrt{4 + 1} = \sqrt{5}
\]

\((1, -2)\) to \((2, 1)\)

\[
\sqrt{(2 - 1)^2 + (1 - (-2))^2} = \sqrt{1 + 9} = \sqrt{10}
\]

Perimeter = \(\sqrt{13} + \sqrt{5} + \sqrt{10}\)

**Circle and Circumference**

- **Radius**
- **Diameter**
- **Circumference**

**Def:** The ratio of the circumference of a circle and its diameter is \(\pi\).
\[ \pi = \frac{\text{circumference}}{\text{diameter}} \]

\[ \text{circumference} = \pi \cdot \text{diameter} \]

\[ (\text{diameter} = 2 \cdot \text{radius}) \]

\[ \text{circumference} = 2\pi \cdot \text{radius} \]

**Example:** Find the radius of a circle that has a circumference of 11 inches.

\[ 11 = 2\pi \cdot \text{radius} \]

Radius = \( \frac{11}{2\pi} \)