If a divides b and b divides c, then a divides c.

Assume a|b and b|c. Then by definition of divides there are integers \( k_1, k_2 \) such that \( ak_1 = b \) and \( bk_2 = c \).

Then \( c = bk_2 = ak_1k_2 \). Since the product of two integers is also an integer, \( ak_1k_2 \) by definition.

**4.2 By contradiction, \( n^2 \text{ odd} \implies n \text{ odd} \)**

Assume for contradiction that there exists an \( n \) such that \( n^2 \) is odd and \( n \) is not odd (so even). Since \( n \) is even, there is an integer \( k \) such that \( n = 2k \).

Then \( n^2 = (2k)^2 = 4k^2 = 2(2k^2) \) and so \( n^2 \) is even, since \( 2k^2 \) is an integer.

So \( n^2 \) is even and \( n^2 \) is odd, a contradiction. Thus, the original statement assumption must be false; there is no \( n \) with \( n^2 \) odd and \( n \) even. Thus, \( n^2 \text{ odd} \implies n \text{ odd} \).

**5.5 If \( a, b \in \mathbb{R} \) and \( n \in \mathbb{Z} \), \( n \geq 0 \), \( \frac{n}{i=0} \frac{1}{(at+b)} = \frac{1}{2} \frac{(n+1)(2a+nb)}{i=0} \)**

The proof will be by induction.

**Base case:** if \( n = 0 \) then \( \frac{0}{i=0} \frac{1}{(at+b)} = at = a \) and \( \frac{1}{2} \frac{(n+1)(2a+nb)}{i=0} = \frac{1}{2} (1)(2a) = a \).

So \( \frac{0}{i=0} \frac{1}{(at+b)} = \frac{1}{2} \frac{(n+1)(2a+nb)}{i=0} \) as required.

**Inductive step:** Suppose \( \frac{k}{i=0} \frac{1}{(at+b)} = \frac{1}{2} \frac{(k+1)(2a+kb)}{i=0} \) for some integer \( k \).

I will show that \( \frac{k+1}{i=0} \frac{1}{(at+b)} = \frac{1}{2} \frac{(k+2)(2a+(k+1)b)}{i=0} \).

\[
\begin{align*}
\frac{k+1}{i=0} \frac{1}{(at+b)} &= \frac{k}{i=0} \frac{1}{(at+b)} + \frac{a+(k+1)b}{i=0} \\
&= \frac{1}{2} \frac{(k+1)(2a+kb)}{i=0} + a+(k+1)b \quad \text{(by assumption)} \\
&= \frac{1}{2} \left[ (k+1)(2a+kb) + 2a + 2(k+1)b \right] \\
&= \frac{1}{2} \left[ 2a(k+1) + kb(k+1) + 2a + 2(k+1)b \right] \\
&= \frac{1}{2} \left[ 2a(k+1) + 2a + k(k+1)(k+2) \right] \\
&= \frac{1}{2} \frac{(k+2)(2a+(k+1)b)}{i=0} \quad \text{as desired.}
\end{align*}
\]

Thus, by induction, \( \frac{n}{i=0} \frac{1}{(at+b)} = \frac{1}{2} \frac{(n+1)(2a+nb)}{i=0} \) for all non-negative integers.