Pearson’s Chi-square Test

\[(Y_1, \ldots, Y_K) \sim \text{Multinomial}(n, p_1, \ldots, p_K) \Rightarrow \sum_{k=0}^{K} \frac{(Y_k - np_k)^2}{np_k} \xrightarrow{L} \chi^2_{K-1}\]

Applications:

1. Parameters known:
   - Goodness-of-fit (Multinomial): check if \((Y_1, \ldots, Y_K)\) follow a certain multinomial distribution.
   - Goodness-of-fit (General): convert \(X_1, \ldots, X_n\) to \((Y_1, \ldots, Y_K)\) through histogram frequency first.

2. Parameters unknown: estimate them first.
   - Test of Independence: \(\hat{p}_{ij} = \hat{p}_i \hat{p}_j\).
   - Goodness-of-fit (More General)

Midterm Review

1. Let \((X_1, \ldots, X_n)\) and \((Y_1, \ldots, Y_m)\) be two Exponential Distribution with parameter \(\lambda_1\) and \(\lambda_2\) respectively. \([f_X(x) = \lambda \exp\{-\lambda x\}]\)

Design LRT for the following testing problem

\[H_0 : \lambda_1 / \lambda_2 = c \text{ versus } H_1 : \lambda_1 / \lambda_2 > c\]

and compute the approximate critical region of 5% significance level.

2. Story omitted. \((16, 398, 3, 225) \sim \text{Multinomial}(642, pq, (1-p)q, p(1-q), (1-p)(1-q))\)
   
   (a) Compute the MLE for \(p\) and \(q\).
   (b) Do Pearson’s goodness of test using \(\hat{p}\) and \(\hat{q}\).