Math 181B Worksheet Week 6

Two Sample Tests

• Unpaired:
  - Data: $X_1, \ldots, X_n \simiid N(\mu_X, \sigma_X^2)$ independent of $Y_1, \ldots, Y_m \simiid N(\mu_Y, \sigma_Y^2)$
  - Test: $H_0 : \mu_X = \mu_Y$ versus $H_1 : \mu_X \neq \mu_Y$
  - Statistic: $\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_X - \mu_Y, \sigma_X^2/n + \sigma_Y^2/m)$

• Paired:
  - Data: $(X_1, Y_1), \ldots, (X_n, Y_n) \simiid N((\mu_X, \mu_Y), \Sigma)$. Dependency allowed in covariance matrix $\Sigma = \begin{pmatrix} \sigma_X^2 & \rho \sigma_X \sigma_Y \\ \rho \sigma_X \sigma_Y & \sigma_Y^2 \end{pmatrix}$.
  - Test: $H_0 : \mu_X = \mu_Y$ versus $H_1 : \mu_X \neq \mu_Y$
  - Statistic: $\bar{X} - \bar{Y} \sim \mathcal{N}(\mu_X - \mu_Y, (\sigma_X^2 + \sigma_Y^2 + 2\rho \sigma_X \sigma_Y)/n)$

• Studentized Statistic: $T = \frac{\bar{X} - \bar{Y}}{se(\bar{X} - \bar{Y})}$
  - Known $\sigma_X^2$ and $\sigma_Y^2$: use directly, $T \sim \mathcal{N}(0,1)$
  - Unknown $\sigma_X^2$ and $\sigma_Y^2$: use unbiased estimator, $T \sim t_{df}$
  - Without Normality: use Central Limit Theorem, asymptotically $\mathcal{N}(0,1)$.

Practice

Suppose we have the following data:

<table>
<thead>
<tr>
<th>Group 1</th>
<th>3</th>
<th>0</th>
<th>5</th>
<th>2</th>
<th>5</th>
<th>5</th>
<th>4</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

1. Assume Normality. Test $H_0 : \mu_X = \mu_Y$ versus $H_1 : \mu_X \neq \mu_Y$ under the following conditions:
   (a) $\sigma_X^2 = 3$, $\sigma_Y^2 = 2$
   (b) $\sigma_X^2 = \sigma_Y^2$
   (c) $\sigma_X^2 \neq \sigma_Y^2$

2. Test $H_0 : \sigma_X^2 = \sigma_Y^2$ versus $H_1 : \sigma_X^2 > \sigma_Y^2$. Comment on the two tests above.

3. If now you learn that they are 10 homework graded twice by different graders, what test would you do?

4. Assume Binomial distribution with $n = 5$. Test $H_0 : p_X = p_Y$ versus $H_1 : p_X \neq p_Y$. 

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