1. Let \( \{p_n\} \) be a sequence of polynomials such that \( p_n \) converges uniformly to a function \( \phi \) on \( C = \{\lambda \in \mathbb{C} \mid |\lambda| = 1\} \). Prove that there exists \( f \in H(\mathbb{D}) \) such that \( p_n \to f \) in \( H(\mathbb{D}) \).

2. Suppose that \( f \) is analytic on \( \mathbb{D} \) with \( |f(z)| \leq 1 \) for all \( z \in \mathbb{D} \). If \( f = 0 \) at the distinct points \( a_1, \ldots, a_n \in \mathbb{D} \), prove the inequality,

\[
|f(z)| \leq \prod_{j=1}^{n} \left| \frac{z - a_j}{1 - \overline{a_j} z} \right|,
\]

for all \( z \in \mathbb{D} \). If \( f \) has a double 0 at \( a_j \) for some \( j \), prove that the inequality is strict for all \( z \in \mathbb{D} \).

3. Let \( \mathcal{F} \) be the collection of analytic functions on \( \mathbb{D} \) whose power series expansion, \( \sum_{n=0}^{\infty} a_n z^n \), satisfies \( |a_n| \leq n \) for all \( n \geq 0 \). Prove that \( \mathcal{F} \) is a normal family.

4. Evaluate

\[
\prod_{n=2}^{\infty} \left( 1 - \frac{1}{n^2} \right)
\]

in the following two ways: (i) directly, and (ii) from the Weierstrass factorization of the \( \sin \) function.

5. Let \( G \) be a connected open set and let \( \{f_n\} \) be a sequence in \( H(G) \). Assume that \( \prod_{n=1}^{\infty} f_n \) converges in \( H(G) \) to a function \( f \) which is not identically 0. Show that for \( a \in G \), \( f(a) = 0 \) if and only if there exists an \( n \) such that \( f_n(a) = 0 \).