Math 109 Homework 1

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1. Prove that \( \{ x \in \mathbb{R} \mid [0,1] \subseteq [0,x) \} = (1, \infty) \).

2. Let \( A, B, C, D \) be sets. Use the Equality Principle to prove that

\[
(A \cup B) \cap (C \cup D) = (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D). \tag{1}
\]

3. Give a proof of (1) using Proposition 1.34 on page 17.

4. Let \( \mathbb{R}^+ = \{ \delta \in \mathbb{R} \mid \delta > 0 \} \). Compute each of the following sets. Be sure to justify your answer.

\[
(i) \quad \bigcup_{\delta \in \mathbb{R}^+} (-\delta, \delta) \quad \text{and} \quad \bigcap_{\delta \in \mathbb{R}^+} (-\delta, \delta)
\]

\[
(ii) \quad \bigcup_{\delta \in \mathbb{R}^+} [\delta, \infty) \quad \text{and} \quad \bigcap_{\delta \in \mathbb{R}^+} [\delta, \infty).
\]