Math 109 Quiz 1 Solution

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Problem. Let 3 sets $A, B$ and $C$ be defined by the formulas $A = (1, \infty)$, $B = (-\infty, 2)$, and $C = \{x \in \mathbb{R} \mid x^2 - 3x + 1 < 0\}$. Show that $A \cap B \subseteq C$.

Solution. That $A \cap B \subseteq C$ will follow from the definition of set inclusion if we can show that every element of $A \cap B$ is an element of $C$. Accordingly, fix $x \in A \cap B$. By the definition of the intersection of two sets, this implies that $x \in A$ and $x \in B$. Since $x \in A$, we have that
\[x \in \mathbb{R} \quad \text{and} \quad 1 < x.\] (1)

Also, since $x \in B$, we have that $x < 2$, or equivalently, that
\[1 < 3 - x.\] (2)

It follows by multiplying the inequalities in statements (1) and (2) that
\[1 < x(3 - x),\]
or equivalently, that
\[x^2 - 3x + 1 < 0.\]

Since $x \in \mathbb{R}$ and $x^2 - 3x + 1 < 0$, $x \in C$. 
