Part 1. Prove that if $f$ and $g$ are bounded real valued functions defined on a set $S \subseteq \mathbb{R}$ and $f(x) \leq g(x)$ for all $x \in S$, then
\[ \inf_{x \in S} f(x) \leq \inf_{x \in S} g(x). \]

Part 2. Use Part 1 to prove that if $f$ and $g$ are integrable functions on $[a, b]$ satisfying $f(x) \leq g(x)$ for all $x \in [a, b]$, then
\[ \int_a^b f \leq \int_a^b g. \]