1. Let \( \{p_n\} \) be a sequence of polynomials such that \( p_n \) converges uniformly to a function \( \phi \) on \( \mathbb{C} = \{ \lambda \in \mathbb{C} | |\lambda| = 1 \} \). Prove that there exists \( f \in H(\overline{D}) \) such that \( p_n \to f \) in \( H(D) \).

2. Let \( G \) be a simply connected region in the plane and assume that \( f \) is a conformal map from \( G \) to \( \overline{D} \) (i.e., \( f : G \to \overline{D} \) is an analytic bijection). Prove that if \( g \) is any other conformal map from \( G \) to \( \overline{D} \) then there exist \( c, \alpha \in \mathbb{C} \) with \( |c| = 1 \) and \( \alpha \in \overline{D} \) such that \( g(z) = c\phi(\alpha f(z)) \).

3. Suppose that \( f \) is analytic on \( \overline{D} \) with \( |f(z)| \leq 1 \) for all \( z \in \mathbb{D} \). If \( f = 0 \) at the distinct points \( a_1, \ldots, a_n \in \mathbb{D} \), prove the inequality,
\[
|f(z)| \leq \prod_{j=1}^{n} \left| \frac{z - a_j}{1 - \overline{a_j}z} \right|
\]
for all \( z \in \mathbb{D} \). If \( f \) has a double 0 at \( a_j \) for some \( j \), prove that the inequality is strict for all \( z \in \mathbb{D} \).

4. Let \( G \) be an open set in \( \mathbb{C} \) and let \( \mathcal{F} \subseteq H(G) \). Prove that if \( \mathcal{F} \) is locally bounded, then \( \mathcal{F} \) is locally Lipschitz, i.e., for each \( a \in G \) there exist \( r > 0 \) and a constant \( M \) such that \( B(a;r) \subseteq G \) and \( |f(z) - f(w)| \leq M|z - w| \) for all \( z, w \in B(a;r) \).

5. Let \( \mathcal{F} \) be the collection of analytic functions on \( \mathbb{D} \) whose power series expansion, \( \sum_{n=0}^{\infty} a_n z^n \), satisfies \( |a_n| \leq n \) for all \( n \geq 0 \). Prove that \( \mathcal{F} \) is a normal family.

6. Evaluate
\[
\prod_{n=2}^{\infty} \left( 1 - \frac{1}{n^2} \right)
\]
in the following two ways: (i) directly, and (ii) from the Weierstrass factorization of the sin function.

7. Let \( G \) be a connected open set and let \( \{f_n\} \) be a sequence in \( H(G) \). Assume that \( \prod_{n=1}^{\infty} f_n \) converges in \( H(G) \) to a function \( f \) which is not identically 0. Show that for \( a \in G \), \( f(a) = 0 \) if and only if there exists an \( n \) such that \( f_n(a) = 0 \).

8. Prove that for each \( \epsilon > 0 \), \( \frac{1}{z+i} + \sin z \) has infinitely many zeros in the region \( \{ z = x + iy | x > 0, |y| < \epsilon \} \).