Problem 1. True or False: For each statement below, determine whether it is true or false, and circle the appropriate letter. You do not need to justify your answer. (5 points each)

(**T** **F**) If A and C are matrices (of the appropriate sizes) and $AC = 0$ then $A = 0$ or $C = 0$.

(**T** **F**) If $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4]$ is a 3×4 matrix and the set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_4\}$ is linearly independent, then the equation $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$ in $\mathbb{R}^3$

(**T** **F**) If A and B are $(n \times n)$ matrices such that A has a column of all zeros, then AB has a column of all zeros.

(**T** **F**) If A is an $m \times n$ matrix and there exist vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ in $\mathbb{R}^n$ such that $A\mathbf{x}_1 = A\mathbf{x}_2$, then for any $\mathbf{b}$ such that the equation $A\mathbf{x} = \mathbf{b}$ has a solution, the solution is not unique.

Problem 2.

Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}$.

a) (20 points) Find a solution to the equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \begin{bmatrix} 3 \\ -1 \\ -6 \\ -2 \end{bmatrix}$.

b) (10 points) Is the solution you found above unique? Why or why not?

Problem 3. (15 points each)

Let $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

a) Find A^{-1}.

b) Use A^{-1} to find the solution to $A\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$

Problem 4. (10 points each)
a) Give an example of a 4×3 matrix A such that the solution set to $A\mathbf{x} = \mathbf{0}$ is a line.

b) Give an example of a 4×3 matrix A such that $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ of the form $\mathbf{b} = \begin{bmatrix} b_1 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}$ and say why your example works.