HW2: MATH 281B

Please work carefully, and show the steps in your calculations, not just the final answer. Your solutions will be graded both for completeness and correctness. Because the department has limited resources available for homework grading, not every problem will be graded. Instead, a selection of problems will be chosen each week to be graded.

Students are encouraged to work in groups for homework assignments. Nonetheless, all submitted work must be individual and verbatim copies are not allowed.

(Liapounov CLT) Suppose X_n is a sequence of independent random variables and that for some $\delta > 0$

$$\frac{1}{s_n^{2+\delta}} \sum_{j=1}^{n} \mathbb{E}|X_j - E(X_j)|^{2+\delta} \to 0 (n \to \infty)$$

with $s_n^2 = \sum_{i=1}^n \mathrm{var}(X_i)$. Then

$$\frac{1}{s_n} \sum_{i=1}^{n} (X_i - \mu_i)$$

converges weakly to standard normal random variable.

(Hajek-Sidak CLT) Let X_i be i.i.d. with mean μ and variance $\sigma^2 < \infty$. Let $c_n = (c_{n1}, \cdots, c_{nn})$ be a vector of constant such that

$$\max_{1 \le i \le n} \frac{c_{ni}^2}{\sum_{j=1}^n c_{nj}^2} \to 0, n \to \infty.$$

Then,

$$\frac{1}{\sigma \sqrt{\sum_{j=1}^n c_{nj}^2}} \sum_{i=1}^{n} c_{ni}(X_i - \mu_i)$$

converges weakly to standard normal random variable.

Solve exercises assigned during lectures and the following exercises.

Problem 1: Suppose $X_1, \cdots, X_n$ are i.i.d. $\mathcal{X}^2(2)$ with density $\frac{1}{2}\exp\{-x/2\}$. Verify that the density of

$$Z_n = \frac{\sqrt{n}(\bar{X} - 2)}{2}$$

Date: Due in-class Thursday, January 30th, 2013 (11:30am).

converges point wise to the density of standard normal random variable. [Hint: use Stirling Formula for approximating log of the Gamma function]

Problem 2: Let X_i be i.i.d., having t-distribution with 2 degrees of freedom. Show, by truncating X_i at $\sqrt{n}\log\log n$ that $b_n\bar{X}$ converges weakly to standard normal random variable. Identify b_n.

Problem 3: Consider a regression problem with

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

with ε_i i.i.d having mean 0 and variance σ^2. [Error does not have Gaussian Distribution] Find conditions on x_i's (using Hajek-Sidak CLT) such that least squares solution has normal distribution.