HW4: MATH 281B

Please work carefully, and show the steps in your calculations, not just the final answer. Your solutions will be graded both for completeness and correctness. Because the department has limited resources available for homework grading, not every problem will be graded. Instead, a selection of problems will be chosen each week to be graded.

Students are encouraged to work in groups for homework assignments. Nonetheless, all submitted work must be individual and verbatim copies are not allowed.

Solve the following exercises.

Problem 1 : Consider a sample $X_1, \cdots, X_n$ to be from normal distribution with unknown μ and σ^2.
Show that the MLE estimator is a consistent estimator of μ, σ^2. [Hint: Consider restriction of the parameter space of this form $\Theta_\varepsilon = \{(\mu, \sigma) : \sigma \geq \varepsilon\}$]

Problem 2 : Suppose $X_1, \cdots, X_n$ are independent observations from the $\text{Bin}(1; \theta^*)$ distribution, for a θ^* with $0 < \theta^* < 1$. Show that MLE estimator $\hat{\theta}_n$ is an M-estimator by finding the function $M_n(\theta)$. Show that

$$\hat{\theta}_n = \theta^* + 0_P(1/\sqrt{n}).$$

Problem 3: Suppose $(X_1, Y_1), \cdots, (X_n, Y_n)$ are observations collected from a Probit Model (a binary regression model), where $X_i \in \mathbb{R}^p$ and $Y_i \in \{-1, 1\}$ with

$$P_\theta(Y_i = 1) = \Phi(\theta^T X_i).$$

Show that the MLE estimator $\hat{\theta}_n$ of θ is an M-estimator. Identify the functions M_n and show it to be consistent estimator of θ.

Problem 4: Consider a linear regression setup with

$$Y_i = \theta^T X_i + \varepsilon_i$$

for ε_i with double exponential distribution. Consider Tukey's bi-weight M-estimator defined by

$$\hat{\theta}_n = \arg\min \frac{1}{n} \sum_{i=1}^{n} \rho(Y_i - \theta^T X_i)$$

with

$$\rho'(x) = \begin{cases} x(1 - (x/\kappa)^2)^2 & \text{if} |x| \leq \kappa \\ 0 & \text{otherwise} \end{cases}$$

Date: Due in-class Thursday, February 25th, 2013 (11:30am).

where $\kappa > 0$ is called "tuning constant". For the purposes of the exercise you can treat κ as known quantity. Show that the estimator $\hat{\theta}_n$ is consistent estimator of θ^* and identify what θ^* is.

Solve the following exercises from TPE book of Lehman.

Problem 6-7: Chapter 1/Section 9/ Problems 8.15, 8.16