Chapter 13: Vector Valued Functions, and Calculus of Vector Valued Functions

THis chapterfocuses on VECTOR-VALUED FUNCTIONS that take in one value (t), and outputs a VECTOR/VECTOR FUNCTION, $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$

Section 13.1: Vector Parametrizations

Two ways given to parametrize a curve:

1. Set one variable to t, and make all other functions into functions of t (NOTE: If you have functions with square roots, then you will likely have TWO parametrizations). See exercise 21 in section 13.1
2. Use trig functions. Example: $x^2 + y^2 = 16$ and $y^2 - x^2 = z - 1$, set $x = 4\cos t$ and $y = 4\sin t$, and solve for z in the second equation

13.2: Calculus of Vector-valued functions

For vector valued functions, a lot of the calculus carries over from single variable calculus to multivariable calculus. You're basically doing the same things, only multiple times.
So let's say:

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle, \quad \mathbf{r}_1(t) = \langle x_1(t), y_1(t), z_1(t) \rangle, \quad \mathbf{r}_2(t) = \langle x_2(t), y_2(t), z_2(t) \rangle$$

And $f(t)$ is a SINGLE VARIABLE SCALAR FUNCTION.

Rule	Vector Valued Version
Derivative	$\mathbf{r}'(x) = \lim_{t\to 0} \frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h} = \langle x'(t), y'(t), z'(t) \rangle$
Integral	$\int \mathbf{r}(t)dt = \langle \int x(t)dt, \int y(t)dt, \int z(t)dt \rangle$
Product rule	$(f(t)\mathbf{r}(t))' = f'(t)\mathbf{r}(t) + f(t)\mathbf{r}'(t)$
DOT product rule	$(\mathbf{r_1}(t) \cdot \mathbf{r_2}(t))' = \mathbf{r_1}'(t) \cdot \mathbf{r_2}(t) + \mathbf{r_1}(t) \cdot \mathbf{r_2}'(t)$
CROSS product rule	$(\mathbf{r_1}(t) \times \mathbf{r_2}(t))' = \mathbf{r_1}'(t) \times \mathbf{r_2}(t) + \mathbf{r_1}(t) \times \mathbf{r_2}'(t)$
Chain rule	$(\mathbf{r}(f(t)))' = \mathbf{r}'(f(t))f'(t)$

13.3: Arc Length Parametrization:

The length of a path for $a \le t \le b$ of $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ is:

$$\int_a^b ||\mathbf{r}'(t)||dt = \int_a^b \sqrt{x'(t) + y'(t) + z'(t)}dt$$

13.5: Motion in Three Space

Given a parametrization $\mathbf{r}(t)$ that traces out a path, we have that the velocity $\mathbf{v}(t)$, acceleration $\mathbf{a}(t)$, and speed $s(t)$ (speed is a single variable function of t) is defined by:

$$\mathbf{v}(t) = \mathbf{r}'(t) \quad \mathbf{a}(t) = \mathbf{r}''(t) \quad s(t) = ||\mathbf{r}'(t)||$$

Given a parametrization for the ACCELERATION $\mathbf{a}(t)$ that gives you the acceleration at time t, we have that the velocity $\mathbf{v}(t)$, position $\mathbf{r}(t)$, and speed $s(t)$ (speed is a single variable function of t) is defined by:

$$\mathbf{v}(t) = \int \mathbf{a}(t) + \langle c_1, c_2, c_3 \rangle \quad \mathbf{r}(t) = \int \mathbf{v}(t) + \langle k_1, k_2, k_3 \rangle \quad s(t) = ||\mathbf{v}(t)||$$

Where $c_1, c_2, c_3, k_1, k_2, k_3$ are all constants of integration. You should know what these are and why you need them when integrating.

And we also have Newton's Second Law:

$$\mathbf{F}(t) = m\mathbf{a}(t)$$

Where $\mathbf{F}(t)$ is the force vector valued function, $\mathbf{a}(t)$ is the acceleration vector valued function, and m is the mass. If you're given the force, then you can find the acceleration by just dividing by the mass.