Announcements:

1. **THE MOST IMPORTANT THING YOU HAVE TO DO IS FILL OUT TA EVALS: (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY FILL OUT:**

   https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx)

   Okay, so I really Really REALLY want you all to fill out your TA Evals. If ANY of the three of us–Me (Jor-el), Daniel, and/or Yan–have done ANYTHING for you–whether it's office hours, discussion sections, review sessions, E-mails, deep, meaningful, philosophical talks, OR ANYTHING, PLEASE PLEASE fill out those evals. We awkward math people need all the help we can get to be better at being NOT awkward math people.

2. **I'LL BE HOSTING A REVIEW SESSION FOR THE FINAL.** Because you are my professor's students, and Ohana means Family and Family means NO ONE GETS LEFT BEHIND. Planned length is up to 4 hours. If you would like to attend, please fill out the poll here: http://www.whenisgood.net/xx48qee

   Let me know when you are able to attend a review session. **PLEASE SELECT ALL OF THE TIMES THAT YOU ARE AVAILABLE TO PARTICIPATE AND I'LL CHOOSE THE TIME WITH THE BEST OVERLAP TO HOST THE REVIEW SESSION.** Please fill out the survey, and do so completely. Under name, please fill in your name and PID. Under comments, include anything you'd like to cover during the review session

3. **A Practice Final, Solutions, and Handout will be made available as soon as I find the time to meet with the professor and ask what to emphasize for the review session.** Details about these things to come. Depending on my schedule, I may also be holding extra office hours before the final as well.

14.7: Optimization in Several Variables (AKA the most tedious section Rogawski has probably ever written)

Important definitions and formulas:

Definition 1. *A function $f(x, y)$ has a **local extremum** at $P = (a, b)$ if there exists an open disk $D(P, r)$ such that:*
Local Maximum: *$f(x, y) \leq f(a, b)$ for all points in the open disk.*
Local Minimum: *$f(x, y) \geq f(a, b)$ for all points in the open disk.*

This means for a **local maximum**, for some small area around the point (a, b), every other point in that area has a SMALLER function value or has the function value equal to $f(a, b)$. For a **local minimum**, for some small area around the point (a, b), every other point in that area has a LARGER function value or has the function value equal to $f(a, b)$.

Definition 2. ***Critical Point:*** *A point $P = (a, b)$ IN THE DOMAIN of f(x,y) is called a* ***Critical Point*** *if*
$f_x(a, b) = 0$ *OR* $f_x(a, b)$ *does not exist*
$f_y(a, b) = 0$ *OR* $f_x(a, b)$ *does not exist*

Why is this important? **Local Maximums and Local Minumums Occur at Critical Points**

Definition 3. ***Saddle Point****: A function $f(x, y)$ has a* ***saddle point*** *at point $P = (a, b)$ if P IS A CRITICAL POINT but it is* ***NOT A MINIMUM OR A MAXIMUM***

Definition 4. ***Discriminant:*** *The* ***discriminant*** *of f(x,y) at a point $P = (a, b)$ is the quantity/number:*

$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$$

Why do you need to know this? **Because you can use it to check if your critical point is a minimum, maximum, or saddle point.**

Second Derivative Test Guide:

Case 1: Discriminant is POSITIVE, and f_{xx} is POSITIVE

$D(a, b) > 0$, $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum

Case 2: Discriminant is POSITIVE, and f_{xx} is NEGATIVE

$D(a, b) > 0$, $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum

Case 3: Discriminant is NEGATIVE

$D(a, b) < 0$, then there is a saddle point at $P = (a, b)$

Case 4: Discriminant is ZERO

$D(a, b) = 0$, then the test fails and is inconclusive; It could be any of the three things above

Here's a handy table for the Second Derivative Test:

Discriminant $D(a, b)$	f_{xx}	Result
Positive, > 0, $+$	Negative, < 0,$-$	$f(a, b)$ is a Local Max
Positive, > 0, $+$	Positive, > 0, $+$	$f(a, b)$ is a Local Min
Negative, < 0,$-$	n/a	Saddle Point at $P = (a, b)$
0	n/a	Test fails/Inconclusive

Example Problem (Global Extreme Values):

14.7.40: Determine the Global Extremes of the function with the following domain (Hey, remember on the second midterm E-mails how I said he won't ask about Domain on the midterm? Well he might ask about it on the final.)

$$f(x, y) = x^3 + x^2y + 2y^2, \quad x, y \geq 0, \quad x + y \leq 1$$

NOTE: The DOMAIN is the triangle enclosed by the three lines.

Step 1: Find the Critical Points

$$f_x(x, y) = 3x^2 + 2xy = x(3x + 2y), \quad f_y(x, y) = x^2 + 4y$$

Set the $f_x = 0 = x(3x + 2y)$, and we get that $x = 0$ or $y = -\frac{3}{2}x$. When $x = 0$, we have $f_y = x^2 + 4y = 0^2 + 4y = 4y = 0$. So $y = 0$ when $x = 0$, and $(0, 0)$ is a critical point.When $y = -\frac{3}{2}x$, we have $f_y = x^2 + 4y = x^2 - 4\frac{3}{2}x = x^2 - 6x = x(x - 6) = 0$. So $x = 0$ when $y = -\frac{3}{2}(0) = 0$, and $(0, 0)$ is a critical point, and $x = 6$ when $y = -\frac{3}{2}(6) = -9$, and $(6, -9)$ is a critical point. Since $y \geq 0$ on the domain, and $-9 < 0$, we have that $(6, -9)$ is NOT in the domain and you dont have to check it. $(0, 0)$, however, is in the domain.

Step 2: Check the Critical Points with the Second Derivative Test

So first we calculate the second order partials.

$$\begin{aligned} f_{xx}(x, y) &= 6x + 2y & \qquad f_{xx}(0, 0) &= 6(0) + 2(0) = 0 \\ f_{yy}(x, y) &= 4 & f_{yy}(0, 0) &= 4 \\ f_{xy}(x, y) &= 2x & f_{xy}(0, 0) &= 2(0) \end{aligned}$$

Next we find the Discriminant:

$$D(0, 0) = f_{xx}(0, 0)f_{yy}(0, 0) - f_{xy}^2(0, 0) = 4(0) - 0^2 = 0$$

Oops, the second derivative tests tells us nothing in this case.

Step 2.5: Find the function values at the critical points

Okay, but we should still find the function values at the critical points. So

$$f(0, 0) = 0^3 + (0)^2(0) + 2(0)^2 = 0$$

Remember this for later

Step 3: Check any endpoints of the boundary

I would do this even if the ook doesn't explicitly say to do this. That's becase when you check the boundary itself, the endpoints could be global extremes when you restrict the function to the boundary. If your boundary is a square or triangle or some other shape with corners, the "endpoints" are the corners. In this case, the corners are $(1, 0)$, $(0, 1)$, and $(0, 0)$. So we find the function values at those points. We already found $f(0, 0)$, so try to find $f(1, 0)$ and $f(0, 1)$.

$$f(1,0) = 1^3 + (1)^2(0) + 2(0)^2 = 1 \quad f(0,1) = 0^3 + (0)^2(1) + 2(1)^2 = 2$$

Remember this for later

Step 4: Check the Boundary and its critical points

So asically what this means is take the functions of the boundary, plug it into the original equation, and find the critial points along the boundary. So we have 3 boundary functions: $x = 0$, $y = 0$, and $x + y = 1$, or $y = -x + 1$.

First set $x = 0$. Then

$$f(x, y) = 0^3 + (0)^2(y) + 2(y)^2 = 2y^2 \quad f_x(x, y) = 0 \quad f_y(x, y) = 4y$$

But this only has critical points along the line $y = 0$. But since this is along the border $x = 0$, we have that the only critical point along this border is $(0, 0)$, and we already checked that.

Next set $y = 0$. Then

$$f(x, y) = x^3 + x^2(0) + 2(0)^2 = x^3 \quad f_x(x, y) = 3x^2 \quad f_y(x, y) = 0$$

But this only has critical points when $x = 0$. But since this is along the border $y = 0$, we have that the only critical point along this border is $(0, 0)$, and we already checked that.

Next set $y = -x + 1$. Then

$$f(x, y) = x^3+x^2(-x+1)+2(-x+1)^2 = x^3-x^3+x^2+2x^2-4x+2 = 3x^2-4x+2 \quad f_x(x, y) = 6x-4 \quad f_y(x, y) = 0$$

Since along this border, f_y is ALWAYS 0, we only need to set f_x to zero to find the critical points along this border. So we have $f_x = 6x - 4 = 0$, or that $x = \frac{2}{3}$. Along the border, $y = -x + 1 = -\frac{2}{3} + 1 = \frac{1}{3}$. FINALLY, we check the value of the function at this point.

$$f(\tfrac{2}{3}, \tfrac{1}{3}) = \tfrac{2}{3}^3 + (\tfrac{2}{3})^2(\tfrac{1}{3}) + 2(\tfrac{1}{3})^2 = \tfrac{8}{27} + \tfrac{4}{27} + \tfrac{2}{9} = \tfrac{12}{27} + \tfrac{6}{27} = \tfrac{18}{27} = \boxed{\tfrac{2}{3}}$$

Remember this.

Step 5: Figure out the Global Min and Max

We have:

$$f(0,0) = 0, \quad f(1,0) = 1, \quad f(0,1) = 2, \quad f(\tfrac{2}{3}, \tfrac{1}{3}) = \tfrac{2}{3}$$

So how do we find the global max and min? Basically just check all the values above; the highest value above would be the global max and the lowest value above would be the global min.

$$\boxed{\text{global min is } f(0,0) = 0 \text{ at point } P = (0,0), \quad \text{global max is } f(0,1) = 2 \text{ at point } P = (0,1)}$$

Final Note: Fill out those TA Evals (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY SUPER MEGA ULTRA FILL OUT:
https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx)