Note: Fill out those TA Evals (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY SUPER MEGA ULTRA FILL OUT:
https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx)

Announcements

1. Extra Office Hours: Monday 4-whenever I have to go (AP& M 6436B).

2. THE FINAL IS 8 QUESTIONS, 9 if you include that first question where you write your name, section, ID, and test version LEGIBLY and CLEARLY (*cough* becausesome-ofyoudon't *cough*). THE FINAL WILL COVER ALL 10 OF THE HOMEWORKS. YES, THAT INCLUDES THAT LAST HOMEWORK YOU ARE NOT TURNING IN. IT MAY ALSO INCLUDE 15.3 MATERIAL.

   Breakdown of the final:

   2 Questions Midterm 1 ish Material

   2 Questions Midterm 2 ish Material

   4 Questions Everything After Midterm 2

   Be expected to be hit with everything up to 15.4, not including the sections the professor told you to skip, and any optional sections. (15.3 may be included though)

   IT WOULD BE A GOOD IDEA TO AT LEAST LOOK AT HOMEWORK 10, AND IT WOULD BE A GREAT IDEA TO DO HOMEWORK 10.

   Needless to say, you're going to need to study for this final, especially for those of y'all who want/need to ace it.

3. OTHER THINGS ABOUT THE FINAL: - Remember when I said you didn't have to worry about domain problems last time? Well now you do. In the context of integration and optimization, DOMAIN IS IMPORTANT. KNOW HOW TO GRAPH DOMAINS AND HOW TO INTERPRET A DOMAIN OF INTEGRATION/OPTIMIZATION.

   Things that won't be on the final:

   - Drawing 3D Graphs

   - Matching Graphs

   Things that will be on the final that you might not have expected:

   - POLAR COORDINATES (11.3), INCLUDING GRAPHING

   - CYLINDRICAL COORDINATES (15.4)

   - TRIPLE INTEGRALS (15.3)

   I've put together a list of recommended problems from these sections that you should take a look at.

4. LOGISTIC DETAILS: BRING YOUR TRITON ID CARD BECAUSE THEY WILL BE CHECKING DURING THE FINAL. Please bring a blue book (or blue books if you prefer), as per usual. You are allowed a single cheat sheet, 8.5"x11", HANDWRITTEN NOT TYPED. NO calculators.

5. TA EVALS ARE DUE SOON (3/16). PLEASE FILL THEM OUT ASAP. THE MOST IMPORTANT THING YOU HAVE TO DO IS FILL OUT TA EVALS: (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY FILL OUT:

   https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx)

Review: Integration by parts

Recall from the product rule, for functions u and v:

$$(uv)' = uv' + u'v = udv + vdu$$

For integration by parts, we integrate both sides of the rule above to get

$$\int (uv)' = \int udv + \int vdu \implies \boxed{\int udv = uv - \int vdu}$$

So we choose two parts of the integral, a u and a dv. Generally, we want to take the hardest part and make it "u" and the rest of it as "dv". A handy way of figuring out which function to choose for u is LIPET:

1. L - Logarithmic functions (ln, log, etc)
2. I - Inverse Trig Functions (arctan, arcsin, arccos, etc)
3. P - Polynomials (x^2, $x^3 + 2x + 1$, etc)
4. E - Exponentials (e^2x, 2^x, etc)
5. T - Trig Functions (sin, cos, tan, etc)

In this case we have whatever is closest the top of the list be u, and make dv everything else that's left. Practice final problem 7 makes use of this rule. Once you choose u and dv, you find du by differentiating u and v by integrating dv. Then plug into the equation and solve.

Review: 11.3: Polar Coordinates

Equations and Formulas

The change of variables formula:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad r = \sqrt{x^2 + y^2}, \quad \theta = \arctan(y/x)$$

(NOTE: The change for θ does not always work)

The area formula:

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} (r(\theta))^2 d\theta$$

The derivative $\frac{dy}{dx}$ in terms of polar coordinates (If $r = f(\theta)$):

$$\boxed{\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}}$$

Tips on graphing in polar coordinates:

1. Remembering how the unit circle works is key to knowing how to plot points in polar coordinates
2. If you're stuck on graphing, just plot points. Then connect the dots smoothly
3. negative radii mean that you go on the other side of the origin (if the angle is π and the radius is negative, the point is on the positive x axis)
4. $r = 0$ means that the point is at the origin

14.7: Optimization in Several Variables (AKA the most tedious section Rogawski has probably ever written)

Definition 1. ***Critical Point:*** *A point $P = (a, b)$ IN THE DOMAIN of f(x,y) is called a* ***Critical Point*** *if*
$f_x(a, b) = 0$ *OR* $f_x(a, b)$ *does not exist*
$f_y(a, b) = 0$ *OR* $f_x(a, b)$ *does not exist*

Why is this important? **Local Maximums and Local Minumums Occur at Critical Points**

Definition 2. ***Saddle Point****: A function $f(x, y)$ has a* ***saddle point*** *at point $P = (a, b)$ if P IS A CRITICAL POINT but it is* ***NOT A MINIMUM OR A MAXIMUM***

Definition 3. ***Discriminant:*** *The* ***discriminant*** *of f(x,y) at a point $P = (a, b)$ is the quantity/number:*

$$D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$$

Why do you need to know this? **Because you can use it to check if your critical point is a minimum, maximum, or saddle point.**

Second Derivative Test Guide:

Case 1: Discriminant is POSITIVE, and f_{xx} is POSITIVE

$D(a, b) > 0$, $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum

Case 2: Discriminant is POSITIVE, and f_{xx} is NEGATIVE

$D(a, b) > 0$, $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum

Case 3: Discriminant is NEGATIVE

$D(a, b) < 0$, then there is a saddle point at $P = (a, b)$

Case 4: Discriminant is ZERO

$D(a, b) = 0$, then the test fails and is inconclusive; It could be any of the three things above

Here's a handy table for the Second Derivative Test:

Discriminant $D(a, b)$	f_{xx}	Result
Positive, > 0, $+$	Negative, < 0,$-$	$f(a, b)$ is a Local Max
Positive, > 0, $+$	Positive, > 0, $+$	$f(a, b)$ is a Local Min
Negative, < 0,$-$	n/a	Saddle Point at $P = (a, b)$
0	n/a	Test fails/Inconclusive

The Process For Finding Global Max/Mins:

A more detailed illustration of this process is in the practice final.

1. Step 1: Find the Critical Points
2. Step 2: Check the Critical Points with the Second Derivative Test
3. Step 2.5: Find the function values at the critical points
4. Step 3: Check any endpoints of the boundary
5. Step 4: Check the Boundary and its critical points
6. Step 5: Figure out the global min and max by comparing the function values at all the endpoints, critical points, and boundary critical points. The max is the greatest function value, the min is the smallest function value.

14.8: Lagrange Multipliers

Single Constraint

Intuition:

We have a function $f(x, y, z)$ or $f(x, y)$ and a constraint curve $g(x, y, z) = 0$ or $g(x, y) = 0$. What we want are the global/local values of f along the constraint g. What do we want to do? CHECK THE GRADIENTS.

The idea is that the gradient of g at EVERY point along the constraint is going to be PERPENDICULAR to the constraint. That's because the constraint is a level curve/surface, and the gradient of a function will alway be perpendicular to its level curves/surfaces.

The gradient of f though, that isn't always perpendicular on the constraint curve. The gradient of f represents THE DIRECTION FOR GREATEST INCREASE for f. So the gradient of f varies along the constraint curve. But something SPECIAL happens when the gradient of the function f is parallel to the gradient of the onstraint g. That means the direction of greatest increase or the direction of greatest decrease would require you to get OFF the constraint curve (there's no direction you can move along the constraint curve to immediately increase/decrease; it's one or the other).

So when the gradient's are PARALLEL, you know you have a local min or local max along the constraint.

Lagrange multiplier formula:

$$\nabla f = \lambda \nabla g$$

This is basically the formula to find where the gradient of f is parallel to the gradient of g; λ is supposed to be some constant. For points where this is true, you have a local max or a local min along the constraint.

The process:

1. Find the gradients and plug it into the formula
2. Set the components of the gradients equal to each other
3. Solve for λ or the variables ; find values of x, y, z, and λ so that the equation is satisfied. One common way is to solve for λ, set the λ of each component equal to each other, and then solve for one variable. Plug that into the constraint equation g and then solve for the remaining variables.
4. Plug the values into the equation f to figure out which points are max/min (in general the greatest points are max, the smallest points are min).

Notes on Lagrange Multipliers

1. When the domain is bounded, and the function is continuous, then there is ALWAYS a global min/max
2. If the professor DOESN'T ask you to prove that the global max/min exists or doesn't exist, you can assume that one does.

Multiple Constraints

NOW, we have a function $f(x, y, z)$ and TWO constraints : $g(x, y, z) = 0$ and $h(x, y, z) = 0$. What we want are the global/local values of f along the constraints g AND h. What do we want to do? CHECK THE GRADIENTS.

The Lagrange multiplier formula becomes:

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

Same idea, but now you have two constraints and a harder equation to solve.

15.1 & 15.2: Double Integrals and Harder Double Integrals

Formula for area ($\mathcal{D}$ is the domain):

$$A = \iint_{\mathcal{D}} dA \quad \Big| \quad dA = dxdy \text{ (Euclidean coordinates)} = rdrd\theta \text{ (Polar coordinates)}$$

When taking integrals of multiple variables, you are HOLDING ALL BUT ONE OF THE VARIABLES CONSTANT. For example, for some function $f(x, y)$, you have that:

$$\int_c^d \int_a^b f(x, y)dydx = \int_c^d \left(\int_a^b f(x, y)dy \right) dx$$

And you're taking the INTEGRAL WITH RESPECT TO y, HOLDING x CONSTANT first. Then you integrate again with respect to x. Also, if you have CONSTANT BOUNDS, you can SWITCH THE ORDER ORINTEGRATION.

$$\int_a^b \int_c^d f(x, y)dxdy = \int_a^b \left(\int_c^d f(x, y)dx \right) dy$$

In this case you're taking the INTEGRAL WITH RESPECT TO x, HOLDING y CONSTANT first. Then you integrate again with respect to y.

Odds are you WON'T get constant bounds, and you'll have to integrate under a harder domain. In which case, te process for that is:

Integrating under a non-constant domain

Normally, when you integrate under a domain like this, you want to switch bounds in a way that makes your life easier. Problem 7 on the practice final goes over this in more detail. But here's the general process:

Process for a double integral with non-constant bounds:

1. GRAPH THE DOMAIN. This will help tremendously.
2. Once you've graphed the domain, figure out which function is the lower bound and which is the upper bound (refer to the table below)– for example, if you're looking at changing the first bound from y to x, figure out which function is to the left–that will be the lower bound in the x direction. Then figure out which function to the right–that will be the upper bound in the y direction. In this example, if you're trying to find the bounds in the x direction, you want to solve for x to get the bounds.
3. Figure out the constant range of the the ther bound–in the example, what values does y range from?
4. Switch bounds
5. Integrate

A handy table for which to function to use as the lower bound and upper bound when integrating in terms of x or y.

Bounds	In terms of x	In terms of y
Upper Bound	Right Function	Upper Function
Lower Bound	Left Function	Lower Function

A more detailed description of this process is outlined in problem 7 on the practice final. REMEMBER TO USE THE RIGHT AREA ELEMENT dA.

15.3 & 15.4: Triple Integrals, Polar Coordinates, and Cylindrical Coordinates

Examples are written out in more detail in problem 8 on the practice final.

Process for finding bounds of a triple integral:

1. Find the bounds in terms of one variable. This variable is typically z, and it would likely be a function of x and y
2. Find the intersection of the z bounds; this and any other bounds will form your simple domain. Then you find the bounds the same way you would in the double integral case.

3. Graph the simple domain, find bounds

Formula for Volume ($\mathcal{W}$ is the domain)

$$V = \iiint_{\mathcal{W}} dV \quad \Big| \quad dV = dxdydz \text{ (Euclidean coordinates)} = rdzdrd\theta \text{ (Cylindrical coordinates)}$$

Process for finding bounds of a triple integral in cylindrical coordinates:

1. Convert two variables AND their bounds of to polar coordinates (these will normally be x and y). This will involve a process just like the one in the double integral section; graph the domain and interpret it in terms of polar coordinates.

2. Convert the bounds in the third variable to polar coordinates (example: if $0 \leq z \leq x$, then that becomes $0 \leq z \leq r\cos\theta$)

Note that you have to use the correct volume element dV.