1 Position, Velocity, Acceleration, and Newton's Law

A drunk **10kg** space cat with a rocket pack is flying dangerously through space, annoying alien scientists that are also trying to fly through space. The alien scientists eventually manage to analyze the space cat's rocket pack, and find that the force exerted by the rocket pack is given by the following formula:

$$\mathbf{F}(t) = \left\langle \frac{-20t}{(t^2+1)^2}, 60t, 20 \right\rangle$$

At a particular moment of time, they also manage to analyze and determine the space cat's exact **velocity** $\langle 1, 10, -2 \rangle$ and **position** $\langle 1, 31, 1 \rangle$ at that point in time. They set these values equal to the **initial velocity** and **initial position**

a) Find the acceleration, velocity, and position of the drunk space cat as functions of time t

b) The alien scientists also prepared an electric field trap for the drunk cat over the entire xy-plane (z = 0). Does the cat ever reach this electric field? If so, find the time at which the cat reaches the electric field, as well as its position, velocity, speed, and acceleration at that time.

c) After dealing with the drunk space cat, the alien scientists realized they missed it dearly, and decided to create a flying robot cat to fill the void in their lives. Find the velocity and acceleration of the cat (as functions in terms of t), given that the robot cat's position is expressed as:

$$\mathbf{r}(t) = \left\langle \arctan(t) + 21, \ln t + 13, \sin t + \pi \right\rangle$$

2 Limits and Continuity

Evaluate the following limits if they exist, otherwise, show that they do not exist.

a) $\displaystyle\lim_{(x,y)\to(0,0)} e^{x+y^2}$

b) $\displaystyle\lim_{(x,y)\to(0,0)} x^4 \sin\left(\frac{1}{x^2+|y|}\right)$

c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$

3 Linear Approximations

Use linear approximation to estimate the value of:

$$\arctan(\ln(1.02)) + 1.02\sin(-0.01) + 42$$

4 Tangent Planes

a) Find the equation of the tangent plane at point $(a, b) = (1, 0)$ on the surface given by:

$$f(x, y) = \arctan(\ln(x)) + x\sin(y) + 42$$

b) Find the equation of the tangent plane at point (1,1,1), on the surface given implicitly by the function:

$$-\arctan(\ln(z)) = \ln(xy) + x^3y^2 - 1$$

5 Harmonic Functions

The Laplace operator Δ is defined by $\Delta(f) = f_{xx} + f_{yy}$ for any function $f(x, y)$. A function f is called **harmonic** if $\Delta(f) = 0$. Is the following function harmonic?

$$f(x, y) = xy + \ln(x^2 + y^2)$$

6 Gradients and Directional Derivatives

Find the directional derivative of the function

$$f(x, y, z) = \arctan(\ln(x)) + \ln(xyz) + x^3y^2z$$

In the direction of $\mathbf{v} = \langle 1, -2, 2\rangle$, at the point $P = (1, 1, 1)$.