1 Dot Product and Cross Product

Let
\[ \mathbf{u} = \langle 2, 5, 1 \rangle \quad \mathbf{v} = \langle 1, 2, -2 \rangle \quad \mathbf{w} = \langle 1, -3, 2 \rangle \]

a) (Angle Between Two Vectors)
Find the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

b) (Area of a Parallelogram)
Find the area of a parallelogram spanned by \( \mathbf{v} \) and \( \mathbf{w} \).

c) (Volume of a Parallelipiped)
Find the volume of a parallelipiped spanned by \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \).

2 Planes

a) (Normal and Point)
Find the Equation of a plane that is normal to the vector \( \mathbf{n} = \langle 3, 1, 8 \rangle \) and passes through the point \( P = (1, 2, 3) \).

b) (Parallel Planes)
Find the equation of a plane that is parallel to the plane \( x + 2y + 3z = 42 \) and passes through the point \( P = (10, 5, 3) \).

c) (Perpendicular Planes)
Find an equation of a plane that is perpendicular to both of the planes found in part a and b.
3  Arc Length and Vector Valued Functions

a) (Eggers’ Problem)
Suppose that for some parametrized curve $r(t)$, we have $r'(t) \cdot r''(t) = 0$. If $r'(0) = \langle 2, 2, 1 \rangle$, find
\[ \int_0^6 \|r'(t)\| dt \]

b) (Easier Arc Length Problem)
Compute the arc length of the curve over the given interval
\[ r(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \leq t \leq 4 \]

4  Directional Derivatives and Gradient

Let
\[ f(x, y) = ye^{y^2-x} \]
And let $P = (1,1)$

a) (Gradient)
Find $\|\nabla f_P\|$, the magnitude of the gradient of $f$ at $P$

b) (Derivative with respect to vector $v$)
Let $v = \langle 1, 2 \rangle$. Find the derivative $D_v f(P)$

c) (Angles and Gradients)
Find the rate of change of $f$ in the direction of a vector making a 45° angle with $\nabla f_P$.

5  Implicit Differentiation

a) (Implicit Partial)
Calculate the partial derivative $\frac{\partial z}{\partial y}$ using implicit differentiation:
\[ e^{xy} + \sin(xz) + y = 0 \]

b) (Implicit Tangent Plane)
Find the equation of the tangent plane at point $P = (1,0,\pi)$. 

c) (Linear Approximation)

Find the formula for the linear approximation $L(x, y)$ using the point $P = (1, 0, \pi)$ for the implicit surface in part a)

6 Global Extremes and Optimization

a) (Triangle Domain)

Determine the Global Extremes of the function with the following domain (Hey, remember on the second midterm E-mails how I said he won’t ask about Domain on the midterm? Well he might ask about it on the final.)

$$f(x, y) = x^3 + x^2y + 2y^2, \quad x, y \geq 0, \quad x + y \leq 1$$

b) (Optimizing with Multiple Constraints)

The cylinder $x^2 + y^2 = 1$ intersects the plane $x + z = 1$ in an ellipse. Find the point on the ellipse that is farthest from the origin.

7 Double Integrals

Compute the integral of $f(x, y) = (\ln y)^{-1}$ Over the domain $D$ bounded by $y = e^x$ and $y = e^{\sqrt{x}}$

8 Triple Integrals

a) (Volume)

Find the volume of the solid in the octant $x \geq 0, \ y \geq 0, \ z \geq 0$ bounded by $x + y + z = 1$ and $x + y + 2z = 1$.

b) (Cylindrical)

Express the following triple integral in cylindrical coordinates, then evaluate.

$$\int_{x=-1}^{1} \int_{y=0}^{\sqrt{1-x^2}} \int_{0}^{x^2+y^2} dz \, dy \, dx$$