

Note: Fill out those TA Evals (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY SUPER MEGA ULTRA FILL OUT:

<https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx>)

## 1 Dot Product and Cross Product

Let

$$\mathbf{u} = \langle 2, 5, 1 \rangle \quad \mathbf{v} = \langle 1, 2, -2 \rangle \quad \mathbf{w} = \langle 1, -3, 2 \rangle$$

### a) (Angle Between Two Vectors)

Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

### b) (Area of a Parallelogram)

Find the area of a parallelogram spanned by  $\mathbf{v}$  and  $\mathbf{w}$ .

### c) (Volume of a Parallelepiped)

Find the volume of a parallelepiped spanned by  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$ .

## 2 Planes

### a) (Normal and Point)

Find the Equation of a plane that is normal to the vector  $\mathbf{n} = \langle 3, 1, 8 \rangle$  and passes through the point  $P = (1, 2, 3)$

### b) (Parallel Planes)

Find the equation of a plane that is parallel to the plane  $x + 2y + 3z = 42$  and passes through the point  $P = (10, 5, 3)$

### c) (Perpendicular Planes)

Find an equation of a plane that is perpendicular to both of the planes found in part a and b.

### 3 Arc Length and Vector Valued Functions

#### a) (Eggers' Problem)

Suppose that for some parametrized curve  $r(t)$ , we have  $r'(t) \cdot r''(t) = 0$ . If  $r'(0) = \langle 2, 2, 1 \rangle$ , find

$$\int_0^6 \|r'(t)\| dt$$

#### b) (Easier Arc Length Problem)

Compute the arc length of the curve over the given interval

$$\mathbf{r}(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \leq t \leq 4$$

### 4 Directional Derivatives and Gradient

Let

$$f(x, y) = ye^{y^2-x}$$

And let  $P = (1, 1)$

#### a) (Gradient)

Find  $\|\nabla f_P\|$ , the magnitude of the gradient of  $f$  at  $P$

#### b) (Derivative with respect to vector $v$ )

Let  $\mathbf{v} = \langle 1, 2 \rangle$ . Find the derivative  $D_{\mathbf{v}}f(P)$

#### c) (Angles and Gradients)

Find the rate of change of  $f$  in the direction of a vector making a  $45^\circ$  angle with  $\nabla f_P$ .

### 5 Implicit Differentiation

#### a) (Implicit Partial)

Calculate the partial derivative  $\frac{\partial z}{\partial y}$  using implicit differentiation:

$$e^{xy} + \sin(xz) + y = 0$$

#### b) (Implicit Tangent Plane)

Find the equation of the tangent plane at point  $P = (1, 0, \pi)$ .

**c) (Linear Approximation)**

Find the formula for the linear approximation  $L(x, y)$  using the point  $P = (1, 0, \pi)$  for the implicit surface in part a)

**6 Global Extremes and Optimization****a) (Triangle Domain)**

Determine the Global Extremes of the function with the following domain (Hey, remember on the second midterm E-mails how I said he won't ask about Domain on the midterm? Well he might ask about it on the final.)

$$f(x, y) = x^3 + x^2y + 2y^2, \quad x, y \geq 0, \quad x + y \leq 1$$

**b) (Optimizing with Multiple Constraints)**

The cylinder  $x^2 + y^2 = 1$  intersects the plane  $x + z = 1$  in an ellipse. Find the point on the ellipse that is farthest from the origin.

**7 Double Integrals**

Compute the integral of  $f(x, y) = (\ln y)^{-1}$  Over the domain  $\mathcal{D}$  bounded by  $y = e^x$  and  $y = e^{\sqrt{x}}$

**8 Triple Integrals****a) (Volume)**

Find the volume of the solid in the octant  $x \geq 0, y \geq 0, z \geq 0$  bounded by  $x + y + z = 1$  and  $x + y + 2z = 1$ .

**b) (Cylindrical)**

Express the following triple integral in cylindrical coordinates, then evaluate.

$$\int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} \int_0^{x^2+y^2} dz dy dx$$