Preliminary notes:

1. Poll for midterm 1 review session is up: **http://whenisgood.net/b3j8pwh** Please click all the times that will work for you. In the name field, please input your name and PID. Under comments, let me know what you want to do in this review session. I have a few ideas in mind, but would like as much input as you are willing to give. ALSO, the link is on my course website: **http://www.math.ucsd.edu/ jbriones/math20c.15w/index.html**. It is under the link "Poll: Midterm 1 Review Session" on the sidebar. Deadline for the poll is Wednesday, January 21st, 2015 at 5PM (or whenever I collect it).

2. This is NOT an exhaustive list of stuff you should know, but you should probably know all of this for exams and homework.

3. I know these formulas are basic and in the textbook, but they do come up a lot in the course, so please know them

4. Speaking of knowing things, you should all memorize or rememorize your unit circles.

12.3: The Dot Product

Some Formulas and Properties to Remember:

The **Dot product** $\mathbf{v} \cdot \mathbf{w}$ of TWO VECTORS

$$\mathbf{v} = \langle a, b, c \rangle, \quad \langle d, e, f \rangle$$

is the SCALAR defined by

$$\mathbf{v} \cdot \mathbf{w} = ad + be + cf$$

In other words, you multiply each corresponding component together, and then you add them up.

Properties of the Dot Product

1. $\mathbf{0} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{0} = \mathbf{0}$

2. **Commutativity**: $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$

3. **Pulling out scalars:** $(\lambda \mathbf{v}) \cdot \mathbf{w} = \mathbf{v} \cdot (\lambda \mathbf{w}) = \lambda(\mathbf{v} \cdot \mathbf{w})$

4. **Distributive Law:** $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
   $(\mathbf{v} + \mathbf{w}) \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{u}$

5. $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$

ALTERNATIVELY! We have another expression for the dot product:

$$\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||||\mathbf{w}|| \cos(\theta)$$

where θ is the angle between the two vectors. So we get that the angle between two (non-zero) vectors is:

$$\theta = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{w}}{||\mathbf{v}||||\mathbf{w}||}\right)$$

Important Notes:

1. The dot product is **commutative**, so ORDER DOES NOT MATTER. I.e. $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$
2. The dot product takes two VECTORS as input and returns a SCALAR. So things like $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$ are generally impossible.
3. **REMEMBER THIS FACT:** For any non-zero vectors $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v} \cdot \mathbf{w} = \mathbf{0} \Leftrightarrow$ The angle θ between $\mathbf{v}$ and $\mathbf{w}$ is $90°$ or $\frac{\pi}{2}$ radians. **In other words, the dot product of two non-zero vectors is** 0 **if and only if the vectors are perpendicular to each other.**

12.4: The Cross Product

The **Cross Product** $\mathbf{v} \times \mathbf{w}$ of TWO VECTORS

$$\mathbf{v} = \langle a, b, c \rangle, \quad \langle d, e, f \rangle$$

is the VECTOR defined by:

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ d & e & f \end{vmatrix} = i \begin{vmatrix} b & c \\ e & f \end{vmatrix} - j \begin{vmatrix} a & c \\ d & f \end{vmatrix} + k \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$
$$= (bf - ce)\mathbf{i} - (af - cd)\mathbf{j} + (ae - bd)\mathbf{k} = \langle bf - ce, cd - af, ae - bd \rangle$$

Some Formulas and Properties to Remember:

Properties of the Cross Product

1. $\mathbf{v} \times \mathbf{v} = \mathbf{0}$
2. **ANTI-Commutativity**: $\mathbf{v} \times \mathbf{w} = -\mathbf{w} \times \mathbf{v}$
3. **Pulling out scalars:** $(\lambda \mathbf{v}) \times \mathbf{w} = \mathbf{v} \times (\lambda \mathbf{w}) = \lambda(\mathbf{v} \times \mathbf{w})$
4. **Distributive Law:** $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$
   $(\mathbf{v} + \mathbf{w}) \times \mathbf{u} = \mathbf{v} \times \mathbf{u} + \mathbf{w} \times \mathbf{u}$
5. $\mathbf{v} \times \mathbf{v} = 0$ IF AND ONLY IF $\mathbf{v} = \lambda \mathbf{w}$ for some scalar λ OR $\mathbf{v}$ or $\mathbf{w}$ are $\mathbf{0}$

Geometric Description of Cross Product

The cross product is the unique vector following three properties

1. It is orthogonal to $\mathbf{v}$ and $\mathbf{w}$
2. $||\mathbf{v} \times \mathbf{w}|| = ||\mathbf{v}||||\mathbf{w}|| \sin(\theta)$ (θ is the angle between the two vectors and $0 \leq \theta \leq \pi$)
3. The vectors $\{\mathbf{v}, \mathbf{w}, \mathbf{v} \times \mathbf{w}\}$ form a right-handed system

NOTE: The dot product has a similar formula, but with cosine instead of sine. DO NOT GET THESE FORMULAS CONFUSED.

What is a right handed system?

Let's say $\mathbf{u} = \mathbf{v} \times \mathbf{w}$. Then look at the (smaller) angle between $\mathbf{v}$ and $\mathbf{w}$. If you can get from $\mathbf{v}$ to $\mathbf{w}$ **counter-clockwise**, then the vector $\mathbf{u}$ points **towards you**. If you can get from $\mathbf{v}$ to $\mathbf{w}$ **clockwise**, then the vector $\mathbf{u}$ points **away from you**. Then the three vectors form what's called a right-handed system.

Important Notes:

1. The cross product is **anti-commutative**, so ORDER MATTERS.
   In other words $\mathbf{v} \times \mathbf{w} \neq \mathbf{w} \times \mathbf{v}$. Instead, $\mathbf{v} \times \mathbf{w} = -\mathbf{w} \times \mathbf{v}$
2. The dot product takes two VECTORS as input and returns a VECTOR. So we can do $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.
3. We can ALSO find the triple product: $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$,
   but we CANNOT do this: $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{u} \cdot \mathbf{w}) \times (\mathbf{v} \cdot \mathbf{w})$ because when you distribute, you end up trying to cross two scalars
   and we CANNOT do this: $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$, because then you would be trying to cross a scalar with a vector