

Note: Fill out those TA Evals (LINK TO TA EVALS THAT YOU SHOULD REALLY REALLY SUPER MEGA ULTRA FILL OUT:

<https://academicaffairs.ucsd.edu/Modules/Evals/default.aspx>)

1 Dot Product and Cross Product

Let

$$\mathbf{u} = \langle 2, 5, 1 \rangle \quad \mathbf{v} = \langle 1, 2, -2 \rangle \quad \mathbf{w} = \langle 1, -3, 2 \rangle$$

a) (Angle Between Two Vectors)

Find the angle between \mathbf{u} and \mathbf{v} .

b) (Area of a Parallelogram)

Find the area of a parallelogram spanned by \mathbf{v} and \mathbf{w} .

c) (Volume of a Parallelepiped)

Find the volume of a parallelepiped spanned by \mathbf{u} , \mathbf{v} , and \mathbf{w} .

2 Planes

a) (Normal and Point)

Find the Equation of a plane that is normal to the vector $\mathbf{n} = \langle 3, 1, 8 \rangle$ and passes through the point $P = (1, 2, 3)$

b) (Parallel Planes)

Find the equation of a plane that is parallel to the plane $x + 2y + 3z = 42$ and passes through the point $P = (10, 5, 3)$

c) (Perpendicular Planes)

Find an equation of a plane that is perpendicular to both of the planes found in part a and b.

3 Arc Length and Vector Valued Functions

a) (Eggers' Problem)

Suppose that for some parametrized curve $r(t)$, we have $r'(t) \cdot r''(t) = 0$. If $r'(0) = \langle 2, 2, 1 \rangle$, find

$$\int_0^6 \|r'(t)\| dt$$

b) (Easier Arc Length Problem)

Compute the arc length of the curve over the given interval

$$\mathbf{r}(t) = \langle 2t, \ln t, t^2 \rangle \quad 1 \leq t \leq 4$$

4 Directional Derivatives and Gradient

Let

$$f(x, y) = ye^{y^2-x}$$

And let $P = (1, 1)$

a) (Gradient)

Find $\|\nabla f_P\|$, the magnitude of the gradient of f at P

b) (Derivative with respect to vector v)

Let $\mathbf{v} = \langle 1, 2 \rangle$. Find the derivative $D_{\mathbf{v}}f(P)$

c) (Angles and Gradients)

Find the rate of change of f in the direction of a vector making a 45° angle with ∇f_P .

5 Implicit Differentiation

a) (Implicit Partial)

Calculate the partial derivative $\frac{\partial z}{\partial y}$ using implicit differentiation:

$$e^{xy} + \sin(xz) + y = 0$$

b) (Implicit Tangent Plane)

Find the equation of the tangent plane at point $P = (1, 0, \pi)$.

c) (Linear Approximation)

Find the formula for the linear approximation $L(x, y)$ using the point $P = (1, 0, \pi)$ for the implicit surface in part a)

6 Global Extremes and Optimization**a) (Triangle Domain)**

Determine the Global Extremes of the function with the following domain (Hey, remember on the second midterm E-mails how I said he won't ask about Domain on the midterm? Well he might ask about it on the final.)

$$f(x, y) = x^3 + x^2y + 2y^2, \quad x, y \geq 0, \quad x + y \leq 1$$

b) (Optimizing with Multiple Constraints)

The cylinder $x^2 + y^2 = 1$ intersects the plane $x + z = 1$ in an ellipse. Find the point on the ellipse that is farthest from the origin.

7 Double Integrals

Compute the integral of $f(x, y) = (\ln y)^{-1}$ Over the domain \mathcal{D} bounded by $y = e^x$ and $y = e^{\sqrt{x}}$

8 Triple Integrals**a) (Volume)**

Find the volume of the solid in the octant $x \geq 0, y \geq 0, z \geq 0$ bounded by $x + y + z = 1$ and $x + y + 2z = 1$.

b) (Cylindrical)

Express the following triple integral in cylindrical coordinates, then evaluate.

$$\int_{x=-1}^1 \int_{y=0}^{\sqrt{1-x^2}} \int_0^{x^2+y^2} dz dy dx$$