Final Exam MATH 20C, SUMMER SESSION I 2017

NAME:

PID: SECTION ID/TIME/TA: SEAT:

DO NOT OPEN THE EXAM UNTIL THE INSTRUCTOR INDICATES EXAM START

The double bars $\|\|$ refer to the norm/magnitude/length of the vectors, as discussed in lecture. The single bars $||$ refer to the absolute value of a scalar.

Instructions:

1. The only notes you may use are a 3"x5" notecard or equally sized piece of paper with notes on them. The use of electronic devices, textbooks, or extra notes (beyond the allowed 3"x5" note card) is not allowed for the exam.
2. Write your name, PID, section, AND seat number on this cover page.
3. Read each question carefully, answer each question completely.
4. Please write a darkly as you can so that scans can capture your answers in full.
5. Indicate below whether you completed a problem on the provided scratch paper, and need us to consider the scratch paper to fully grade your exam. If you do not check "yes" we will assume that you do not need us to consider the scratch paper to fully grade your exam.
6. Show ALL of your work for each part. You are allowed to refer to your answers on previous parts of a question to solve a part in the same question. **CREDIT MAY NOT BE GIVEN FOR UNSUPPORTED ANSWERS**

Did you complete a problem on the provided scratch paper?

☐ YES ☐ NO

This exam has 8 questions, plus 1 extra credit question. Questions 6, 7, and 8, and the extra credit problem has just one (1) part. Questions 1, 2, 3, and 4 have two (2) parts. Question 5 has three (3) parts. The exam has content on both the front and back of sheets. In total, the exam is 14 pages, were each side of a sheet is a page. The first page is this cover page, and the last 3 pages (1.5 sheets) is scratch paper.

Problem 1. (10 points)

a) Find an equation of the line that passes through the point $(4, 2, 0)$ and is parallel to the line of intersection between the planes $x - 3z = 5$ and $x + 2y = 1$.

b) Find an equation for the plane that contains the point $(5, 4, -3)$ and is perpendicular to the line $l(t) = (\pi, \pi, \pi) + t(8, 5, 17)$.

Problem 2. (10 points)
a) Find all pairs of b and c where $(b, 2, c)$ is orthogonal to both $(2b, 3, -2)$ and $(-10, 3, 1)$.

b) Let $\vec{u}$ and $\vec{v}$ be vectors where $\|\vec{u}\| = 5$, $\|\vec{v}\| = 6$, and $\|\vec{u} - \vec{v}\| = 4$. Find $\|(\vec{u} - \vec{v}) \times (2\vec{u} + \vec{v})\|$. Note: For any scalar k and vector $\vec{w}$, $\|k\vec{w}\| = |k|\|\vec{w}\|$.

Problem 3. (10 points)
a) Suppose $c : \mathbb{R} \to \mathbb{R}^3$ is a path, in other words, $c(t) = (x(t), y(t), z(t))$ for some scalar functions $x(t)$, $y(t)$, and $z(t)$. Suppose also that $c(t)$ is of class C^2, in other words, it is continuous and twice differentiable for all t. If $c'(t) \cdot c''(t) = 0$ for all t, then show that $\|c'(t)\|$ is constant.

b) A particle with a mass of 10 kg is traveling in space. The force acting on the particle is given by

$$F(t) = (50 \cos t, 130 \sin t, -120 \cos t)$$

and the particle has initial velocity $v(0) = (12, -13, 5)$. Find the length of the path traveled by the particle from $t = \frac{\pi}{15}$ to $t = \frac{16\pi}{15}$

Problem 4. (10 points)

a) Let a surface S_1 be defined by:

$$\sin(xz) + e^{xyz} + x^2 y + 2\arctan z = 3$$

Find the equation for the tangent plane to the surface S_1 at the point $(1, 2, 0)$

b) Let a surface S_1 be defined as in part a). Let P be the tangent plane to the surface S_1 at the point $(1, 2, 0)$. Let S_2 be the surface defined by:

$$-x^2 + y + z^5 = 4$$

Find all the points on S_2 where the tangent plane to S_2 is parallel to plane P.

Problem 5. (10 points) A beautiful princess named Jen is skipping toward her castle on top of a hill when Zapdos, the legendary thunder bird, appears at the castle in the direction of greatest increase, to claim the hill as its territory. The altitude, in meters, of the hill is given by

$$H(x, y) = \frac{400}{(x-2)^2 + 2(y-3)^2 + 2}$$

where x, y, and the altitude are measured in meters.

a) Jen is standing at point $(6, 2)$ and is attempting to flee from the thunder bird by hurrying to the base of the mountain in the direction of greatest decrease. What is the direction of greatest decrease? Express the direction of greatest decrease as a unit vector.

b) If Jen is moving at 2 meters per second from the point $(6, 2)$, what is her rate of change if she moves in the direction of greatest decrease?

c) A split second before she actually starts running, Jen remembers that she ate a magic rubber fruit, and that she is immune to electrical attacks. She also remembers that she has a legendary bird repellent to fight the bird.

Still standing at the point $(6, 2)$, she decides to charge toward the thunder bird. However, if her altitude is increasing at a rate greater than $4\sqrt{10}$ meters per second, then she succumbs to mountain sickness, and can no longer defend her territory.

If Jen is moving at 2 meters per second, from the point $(6, 2)$, what is the minimum angle from the direction of greatest increase (at $(6, 2)$) that Jen can move in order to prevent getting mountain sickness?

Problem 6. (10 points)

Find the critical points of $f(x, y) = x^2 - 6xy - 12x + 8y^2 + 18y + 9001$ and determine whether they are local minima, local maxima, or saddle points.

Problem 7. (10 points) Consider the function $f(x, y) = x^2 - 5xy + y^2 + 5$ defined on the disc of radius $\sqrt{2}$, namely, $D = \{(x, y) \mid x^2 + y^2 \leq 2\}$. Find the maximum and minimum points of f on D, and find the maximum and minimum values of f on D.

Problem 8. (10 points) Evaluate the following integral:

$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx dy$$

"The limit does not exist!" – Cady Heron, *Mean Girls* (2004)
"The only limit is your imagination." – Hiro Hamada, *Big Hero 6* (2014)

Problem [Extra Credit]. (1 point)

Prove that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{\ln(x^2+y^2+1)}$$

SCRATCH PAPER

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