Midterm I MATH 20C, SUMMER SESSION I 2017

NAME:

PID: SECTION ID/TIME/TA: SEAT:

DO NOT OPEN THE EXAM UNTIL THE INSTRUCTOR INDICATES EXAM START

The double bars $\|\|$ refer to the norm/magnitude/length of the vectors, as discussed in lecture.

Instructions:

1. The only notes you may use are a 3"x5" notecard or equally sized piece of paper with notes on them. The use of electronic devices, textbooks, or extra notes (beyond the allowed 3"x5" note card) is not allowed for the exam.
2. Write your name, PID, section, AND seat number on this cover page.
3. Read each question carefully, answer each question completely.
4. Indicate below whether you completed a problem on the provided scratch paper, and need us to consider the scratch paper to fully grade your exam. If you do not check "yes" we will assume that you do not need us to consider the scratch paper to fully grade your exam.
5. Show ALL of your work for each part. You are allowed to refer to your answers on previous parts of a question to solve a part in the same question. **CREDIT MAY NOT BE GIVEN FOR UNSUPPORTED ANSWERS**

Did you complete a problem on the provided scratch paper?

☐ YES ☐ NO

This exam has 5 questions. Questions 1, 2, 3, and 5 have two (2) parts. Question 4 has three (3) parts. The exam has content on both the front and back of sheets. In total, the exam is 10 pages, were each side of a sheet is a page. The first page is this cover page, and the last 3 pages (1.5 sheets) is scratch paper.

Problem 1. (10 points)
a) Find the equation of the line of intersection between the planes $x + 2y + 3z = 5$ and $x + 2y - z = 1$

b) Find the equation of a plane that contains the line $q(t) = (7, 2, 5) + t(2, 4, 0)$ and contains the point of intersection between the two lines $r(t) = (5, 7, 9) + t(4, 5, 6)$ and $s(t) = (3 - 2t, 2, 1 + 2t)$

Problem 2. (10 points) Let $\vec{u}$ and $\vec{v}$ be vectors where $\|\vec{u}\| = 5$, $\|\vec{v}\| = 6$, and $\|\vec{u} - \vec{v}\| = 9$.
a) Find $\|2\vec{u} + \vec{v}\|$.

b) Find $\|\vec{u} \times \vec{v}\|$

Problem 3. (10 points) Let the acceleration of a particle be represented by $a(t) = (-4\cos t, 3\cos t, -5\sin t)$. Let the initial velocity be $v(0) = (3, 4, 5)$, and let the initial position on the path be $r(0) = (4, 0, 1)$

a) Find $r(t)$, the equation for the path described above.

b) Find the length of the path $r(t)$ from $t = \frac{\pi}{12}$ to $t = \frac{13\pi}{12}$.

Problem 4. (10 points) Let $f(x, y) = x^2 y$, and let $g(x, y) = x^4 + y^2$.
a) Find

$$\lim_{(x,y)\to(0,0)} \frac{f(x, y)}{g(x, y)}$$

if it exists. Otherwise, if the limit does not exist, show that the limit does not exist.

b) Let P be the tangent plane to $g(x, y)$ at the point $(1, 2, 5)$. Find all points on the graph of f that have tangent plane parallel to P.

c) Using linear approximation to approximate the function g(x,y), estimate $(1.0012)^4 + (2.005)^2$

Problem 5. (10 points) A princess named Jen is walking around a flat field when Moltres, a legendary fire bird appears to drastically change the temperature of the field. The new temperature, in Celsius, of the field is given by

$$T(x,y) = \frac{100}{(x-2)^2 + 4(y-1)^2 + 2} + 30$$

where x and y are measured in meters.
a) Jen is standing at point $(4,2)$ and is attempting to flee from the heat. If Jen is moving at unit speed (1 meter per second) from the point $(4,2)$, in what direction should she run if she wants to run in the direction of fastest temperature decrease? Express the direction of decrease as a vector. What is the directional derivative along the direction of fastest temperature decrease?

b) After fleeing, Jen returns with a heat-resistant suit and enough water to fight the fire bird. She is dropped at the point (0,0) in search of the fire bird. Unfortunately, the heat-resistant suit does not like change and will disintegrate if it is heated at a rate greater than $2\sqrt{15}$ degrees per second, or if it is cooled down at a rate greater than $2\sqrt{5}$ degrees per second.

If Jen is moving at unit speed (1 meter per second), from the point (0,0), what is the minimum angle and maximum angle from the gradient that Jen can move in order to prevent the suit from disintegrating?

SCRATCH PAPER

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