**Practice Final Exam** MATH 20C, SUMMER SESSION I 2017

NAME:

PID: SECTION ID/TIME/TA: SEAT:

# DO NOT OPEN THE EXAM UNTIL THE INSTRUCTOR INDICATES EXAM START

The double bars $\|\|$ refer to the norm/magnitude/length of the vectors, as discussed in lecture.

## Instructions:

1. The only notes you may use are a 3"x5" notecard or equally sized piece of paper with notes on them. The use of electronic devices, textbooks, or extra notes (beyond the allowed 3"x5" note card) is not allowed for the exam.
2. Write your name, PID, section, AND seat number on this cover page.
3. Read each question carefully, answer each question completely.
4. Please write a darkly as you can so that scans can capture your answers in full.
5. Indicate below whether you completed a problem on the provided scratch paper, and need us to consider the scratch paper to fully grade your exam. If you do not check "yes" we will assume that you do not need us to consider the scratch paper to fully grade your exam.
6. Show ALL of your work for each part. You are allowed to refer to your answers on previous parts of a question to solve a part in the same question. **CREDIT MAY NOT BE GIVEN FOR UNSUPPORTED ANSWERS**

## Did you complete a problem on the provided scratch paper?

☐ YES ☐ NO

This exam has 5 questions. Questions 1, 2, 3, and 5 have two (2) parts. Question 4 has three (3) parts. The exam has content on both the front and back of sheets. In total, the exam is 10 pages, were each side of a sheet is a page. The first page is this cover page, and the last 3 pages (1.5 sheets) is scratch paper.

**Problem 1.** (10 points)
a) Find the equation of the line that is parallel to the line $r(t) = (3, 4, 5) + t(5, 4, 3)$ and passes through the point $(1, 2, 3)$.

b) Find the equation of a plane that contains the lines $q(t) = (6, 2, 7)+t(2, 0, 1)$ and $l(t) = (6, 2, 7)+t(0, 3, 2)$.

**Problem 2.** (10 points)

a) Find all values of $b$ where $(b, 12)$ and $(3, 2)$ are orthogonal.

b) Let $\vec{u}$ and $\vec{v}$ be vectors where $\|\vec{u}\| = 5$, $\|\vec{v}\| = 6$, and $\|\vec{u} + \vec{v}\| = 9$. Find $\|\vec{u} \times \vec{v}\|$.

**Problem 3.** (10 points)
a) Suppose $c(t)$ represents the position of a particle at time $t$. Show that if $\|c(t)\|$ is constant, then the position vector $c(t)$ is always perpendicular to the velocity vector $c'(t)$ at any time $t$.

b) Let $c(t) = (\cos t, \sin t, \sqrt{20}t^2)$ represent the position of a particle at time $t$. Find the equation of the tangent line at time $t = \pi$.

**Problem 4.** (10 points) Let $f(x, y) = x^2y$, and let $g(x, y) = x^4 + y^2$.
a) Find an equation for the tangent plane to $g(x, y)$ at the point $(1, 2, 5)$

b) Let $P$ be the tangent plane to $g(x, y)$ at the point $(1, 2, 5)$. Find all points on the graph of $f$ that have tangent plane parallel to $P$. These points will have the format $(a, b, f(a, b))$

**Problem 5.** (10 points) Captain Ishmael Pogo is sailing the seven seas in search of Lugia, the great white legendary bird. Captain Pogo knows that Lugia travels only in the coldest waters, and is searching continuously in the coldest parts of the water. The temperature in Celsius of the water is given by

$$T(x, y) = -20e^{-x^2-2y^2} + 20$$

where $x$ and $y$ are measured in meters. Captain Pogo is currently at $(1, 1)$

a) In what direction should Captain Pogo proceed in order for the temperature to decrease most rapidly? Express your answer as a unit vector.

b) If Captain Pogo is traveling at $e^4$ meters per second, what is the rate of change of the temperature if he proceeds in the direction of greatest decrease?

c) The ship will crack if the water temperature is decreasing at a rate greater than $20e\sqrt{15}$ degrees per second. If Captain Pogo is still traveling at $e^4$ meters per second, what is the maximum angle from the gradient that Captain Pogo can travel to avoid cracking he ship because of the temperature change of the water?

**Problem 6.** (10 points)

a) Find all critical points of the function $f(x, y) = x^2 + y^2 - 3xy$, and use the second derivative test to determine whether they are local maxima, local minima, or saddle points.

b) Use the method of Lagrange Multipliers to determine the maximum and minimum points of $f(x, y) = x^2 + y^2 - 3xy$ in the circle $x^2 + y^2 \leq 1$

**Problem 7.** (10 points) Evaluate the following integral:

$$\int_0^1 \int_y^1 \sin(x^2) dx dy$$

**SCRATCH PAPER**

**SCRATCH PAPER**

**SCRATCH PAPER**