Solving Linear Systems (1.1-1.2)

Terms you should probably know

inconsistent: A linear system is called *inconsistent* if it has no solution.

consistent: A linear system is called *consistent* if it has AT LEAST ONE solution.

leading term: The *leading term* of a row of a matrix is the first non-zero entry in that row.

row equivalent: If a system S_2 is obtained from a system S_1 by elementary row operations, we say that the systems S_2 and S_1 are *row equivalent*.

pivot: Given a matrix A, an entry in A that corresponds to a leading 1 in rref(A) is called a *pivot*.

pivot position: Given a matrix A, the position of an entry in A that corresponds to a leading 1 in rref(A) is called a *pivot position*.

pivot column: A *pivot column* is a column of a matrix that contains a pivot

ALSO: You should know what a coefficint matrix and augmented matrix are, what a system is, and how those things are different and related.

List of Elementary Row Operations

1. Add a multiple of one row to another row
2. Multiply one row by a *non-zero* number
3. Exchange two rows

What is Echelon Form and Reduced Row Echelon Form?

A Matrix in Echelon form is a matrix with the following properties:

1. All the non-zero rows are above zero rows
2. Each leading entry will be in a column to the right of the leading entry of the row above it
3. All entries below a leading entry are zeroes

In addition to that, if the matrix has the following two properties:

1. Each leading entry is a 1
2. The entries above each leading 1 are 0's

then we say that the Matrix is in **Reduced Row Echelon Form**.

NOTE: You should absolutely know how to get from any matrix to its Echelon Form or its Reduced Row Echelon Form using elementary row operations. The Echelon form of a matrix will be very useful for a large portion of this class.

Important Theorems you should know

Theorem. *(Chapter 1, Theorem 2) If an Echelon Form of the augmented matrix of a system has a row of the form:*

$$\left[\, 0\ 0\ 0\ \dots\ 0 \mid x \,\right]$$

where $x \neq 0$, then the system has no solution (aka, the system is ***inconsistent****). Otherwise, the system is has at least one solution (aka, the system is* ***consistent****). If the system has no free variables, then the system has one unique solution.*

To break down the above: If the AUGMENTED MATRIX corresponding to the linear system has the above property, the linear system is inconsistent. Otherwise, the system is consistent. If the system is consistent AND has no free variables, then the system has ONE UNIQUE SOLUTION.

Theorem. *Two row equivalent systems have the same solution set.*