**Math 20F**
**Summer Session 1 2015**
**Practice Final Exam**

**Name (Print):** _____

**Time Limit: 3 Hours**

Teaching Assistant _____

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This exam contains 10 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Work on this practice final ONLY AFTER you have reviewed the homework and lecture problems**. There could be mistakes and the problems could be more or less difficult than the actual final exam questions.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| Total: | 90 | |

1. (10 points) Find a least-squares solution of $Ax = b$ where:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

2. (10 points) Show that if $x$ is an eigenvector of the matrix product $AB$ and $Bx \neq 0$, then $Bx$ is an eigenvector of $BA$.

3. This question concerns diagonalizable matrices.

   (a) (5 points) Is the matrix $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ diagonalizable? If so, explain why and diagonalize it. If not, explain why not.

   (b) (5 points) Is the matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ diagonalizable? If so, explain why and diagonalize it. If not, explain why not.

4. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

   (a) (5 points) Find the eigenvalues and eigenvectors of $A$.

   (b) (5 points) Find the solution to the system of differential equations $x'(t) = Ax(t)$ where $x(0) = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$

5. Consider the matrix $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$ and assume that it is row equivalent to the matrix $B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (5 points) List rank $A$ and dim Nul $A$.

(b) (5 points) Find bases for Col $A$ and Nul $A$. Find a simple example of a vector that belongs to Col $A$, as well as an example of a vector that belongs to Nul $A$.

6. (a) (5 points) Consider an $n \times n$ matrix $A$ with the property that the row sums all equal the same number $s$. Show that $s$ is an eigenvalue of $A$. [Hint: find an eigenvector]

(b) (5 points) Explain or demonstrate that the eigenspace of a matrix $A$ corresponding to some eigenvalue $\lambda$ is a subspace.

7. (a) (5 points) Suppose a $6 \times 8$ matrix $A$ has four pivot columns. What is dim Nul $A$? Is Col $A = \mathbb{R}^4$? Why or why not?

(b) (5 points) If $A$ is a $7 \times 5$ matrix, what is the smallest possible dimension of Nul $A$?

8. (10 points) Find the dimension of the subspace spanned by the given vectors. Are the vectors linearly independent?

$$\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}3\\1\\1\end{bmatrix}, \begin{bmatrix}-2\\-1\\1\end{bmatrix}, \begin{bmatrix}5\\2\\2\end{bmatrix}$$

9. (10 points) Choose two statements at random from the list of 12 problems and re-work them...